Extrait du Bulletin de la Société d'Histoire naturelle de Toulouse, t. 68, 1934.

LA VIGNE ET PRODUCTION DU SUCRE DE RAISIN

PAR

M. G. BRUNEL



Lyell has well remarked, will have recorded only a few of its changes of form. Only a few species or groups of species will have left descendants, and these descendants will form a closely allied group. Only a few forms will have been preserved in any great thickness and extent of sedimentary deposits, which will have endured for a long period, whilst the intervening spaces will have been times of comparative non-deposition. As species are produced and exterminated by slowly acting causes, most geological formations will be discontinuous in their organic contents. The formations which are rich in fossils throughout their whole thickness will rarely be found extending for great distances, for this implies that the sea has remained deep during the whole period of deposition, and that the bottom has not been disturbed; but this implies continuous subsidence, which must be balanced by the rate of accumulation of the deposits to keep the water shallow and the fossils distinct; and such a balance will rarely occur for great distances and for great periods. Where sedimentary deposits are not very thick, or have been intermittent in their accumulation, they will rarely include many fossils, and these will be liable to be obliterated by subsequent metamorphic action. Blank spaces will occur in our geological records, where we find no trace of the species which existed during the intervals between the successive formations; and these intervals may have been of short or long duration.

Where we find a formation containing many species in a recent state, we may feel sure that a record of many species which were living at the time of its deposition, but which have become extinct before the present day, will be preserved in the formation in a fossil state. Species which have a wide range, both formerly and at the present day, will often have produced the largest number of varieties, which will often at first have been recorded as local species; and these varieties will afterwards become converted into species, which will at first be widely diffused, but will finally become exterminated by the victory of the more improved descendants. But we may feel sure that the species which are most widely diffused, both in their own country and throughout the world, will have left the greatest number of modified descendants, which will also in the long run become widely diffused and victorious. Hence the species which have left many descendants, which will be the most successful species, and which will be the most widely diffused, will often be represented by fossils in the greatest number of the successive strata. Species which have had a very restricted range will also, it is probable, have left descendants which will have become modified and will be represented by fossils in the same manner; and this will have been the case with species which have formed the connecting links between many extinct genera and families. Species which have left no descendants, but which have become completely extinct, will be represented by fossils alone; and these will be found in the most ancient formations, and will be of the most aberrant types.

It is a common opinion that the geological record is extremely imperfect, and this will readily be admitted. But if we look to distant parts of the world, we have no reason to believe that one formation is generally more complete than another, or that the average interval between successive formations is greater than a few million years. Although the lapse of time has been immense, it may not have sufficed for the accumulation of sedimentary deposits of sufficient thickness, and sufficiently extended in area, to receive and preserve the remains of all the species which have existed during the successive periods. Many species which have lived during the earlier periods may have become extinct before any sedimentary deposits were laid down in which fossils could be preserved. Of the species which have lived during the later periods, only a few will have left descendants which will have become modified and will be represented by fossils in the same manner; and this will have been the case with species which have formed the connecting links between many extinct genera and families. Species which have left no descendants, but which have become completely extinct, will be represented by fossils alone; and these will be found in the most ancient formations, and will be of the most aberrant types.

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Lord Bacon's Works, and the Character of his Writings.

Lord Bacon's Works, collected and published by Mallet, in 4 vols. folio, 1740, contain, 1. The Philosophical Works, viz. the Novum Organum, the Advancement of Learning, the Sylva Sylvarum, the New Atlantis, and the Natural History of Life and Death. 2. The Moral, Political, and Historical Works, viz. the Essays, the Wisdom of the Ancients, the Reign of King Henry VII. the Life of King Henry VIII. the Life of Queen Elizabeth, the Life of King James, the Life of Edward the Black Prince, the Life of Henry the Fifth, the Life of Henry the Sixth, the Life of Edward the Fifth, the Life of Richard the Third, the Life of Edward the Sixth, the Life of Queen Mary, the Life of Julius Cæsar, the Life of Augustus Cæsar, the Life of Henry the Seventh,

Lemma 1 (Lemma 1 of [1]). Let $\{\mathcal{F}_n\}_{n\geq 0}$ be a filtration, $\{X_n\}_{n\geq 0}$ be a sequence of random variables, and $\{\alpha_n\}_{n\geq 0}$ be a sequence of positive numbers. Suppose that X_n is $\mathcal{F}_n$-measurable, $\mathbb{E}[X_n] = 0$, and $\mathbb{E}[X_n^2] < \infty$ for all $n \geq 0$. Then, on the event $\{\sum_{n=0}^{\infty} \alpha_n^2 \mathbb{E}[X_n^2 | \mathcal{F}_{n-1}] < \infty\}$, we have $\sum_{n=0}^{\infty} \alpha_n X_n < \infty$ almost surely.

Lemma 2 (Lemma 2 of [1]). Let $\{\mathcal{F}_n\}_{n\geq 0}$ be a filtration, $\{X_n\}_{n\geq 0}$ be a sequence of random variables, and $\{\alpha_n\}_{n\geq 0}$ be a sequence of positive numbers. Suppose that X_n is $\mathcal{F}_n$-measurable, $\mathbb{E}[X_n] = 0$, and $\mathbb{E}[X_n^2] < \infty$ for all $n \geq 0$. Then, on the event $\{\sum_{n=0}^{\infty} \alpha_n^2 \mathbb{E}[X_n^2 | \mathcal{F}_{n-1}] < \infty\}$, we have $\sum_{n=0}^{\infty} \alpha_n X_n < \infty$ almost surely.

Lemma 3 (Lemma 3 of [1]). Let $\{\mathcal{F}_n\}_{n\geq 0}$ be a filtration, $\{X_n\}_{n\geq 0}$ be a sequence of random variables, and $\{\alpha_n\}_{n\geq 0}$ be a sequence of positive numbers. Suppose that X_n is $\mathcal{F}_n$-measurable, $\mathbb{E}[X_n] = 0$, and $\mathbb{E}[X_n^2] < \infty$ for all $n \geq 0$. Then, on the event $\{\sum_{n=0}^{\infty} \alpha_n^2 \mathbb{E}[X_n^2 | \mathcal{F}_{n-1}] < \infty\}$, we have $\sum_{n=0}^{\infty} \alpha_n X_n < \infty$ almost surely.

Lemma 4 (Lemma 4 of [1]). Let $\{\mathcal{F}_n\}_{n\geq 0}$ be a filtration, $\{X_n\}_{n\geq 0}$ be a sequence of random variables, and $\{\alpha_n\}_{n\geq 0}$ be a sequence of positive numbers. Suppose that X_n is $\mathcal{F}_n$-measurable, $\mathbb{E}[X_n] = 0$, and $\mathbb{E}[X_n^2] < \infty$ for all $n \geq 0$. Then, on the event $\{\sum_{n=0}^{\infty} \alpha_n^2 \mathbb{E}[X_n^2 | \mathcal{F}_{n-1}] < \infty\}$, we have $\sum_{n=0}^{\infty} \alpha_n X_n < \infty$ almost surely.

Lemma 5 (Lemma 5 of [1]). Let $\{\mathcal{F}_n\}_{n\geq 0}$ be a filtration, $\{X_n\}_{n\geq 0}$ be a sequence of random variables, and $\{\alpha_n\}_{n\geq 0}$ be a sequence of positive numbers. Suppose that X_n is $\mathcal{F}_n$-measurable, $\mathbb{E}[X_n] = 0$, and $\mathbb{E}[X_n^2] < \infty$ for all $n \geq 0$. Then, on the event $\{\sum_{n=0}^{\infty} \alpha_n^2 \mathbb{E}[X_n^2 | \mathcal{F}_{n-1}] < \infty\}$, we have $\sum_{n=0}^{\infty} \alpha_n X_n < \infty$ almost surely.

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Ferdinand Lassalle, Gesammelte Reden und Schriften, Band 1, Berlin 1919, S. 1–2.

Ferdinand Lassalle, Gesammelte Reden und Schriften, Band 1, Berlin 1919, S. 3–4.

Table 1. Statistics of CIFAR-10

Class	0	1	2	3	4	5	6	7	8	9
Train	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000
Test	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000


Ordovician
Silurian
Devonian
Carboniferous
Permian
Triassic
Jurassic
Cretaceous
Paleogene
Neogene
Quaternary
Paleozoic
Mesozoic
Cenozoic
Phanerozoic
Proterozoic
Archean
Hadean
Precambrian

Mr. Bingley was obliged to go to town almost as soon as he was married; and it was impossible that his bride should not be very unwilling to quit Netherfield, as long as Jane remained there. Mrs. Bingley's affection for her sister, and her gratitude to herself, made her exert herself so successfully, that Mr. Bingley's departure was deferred for a month after his marriage. Before he left the country, he had the pleasure of seeing Mrs. Hurst settled in a house of her own, within a few miles of Pemberley; and Mr. Hurst, who had been extremely anxious to be admitted into the family of a man of fortune, was happy enough to be able to visit his wife at Mr. Darcy's invitation, and to consider himself as quite belonging to the family.

Mr. Bingley was extremely attached to his friend, and though he had now begun to live for himself, he could not bear to be separated from him entirely. Darcy's influence over him was unbounded; and he was glad to be able to show his regard for him by placing him in the midst of his own family. Mr. Bingley's fortune was sufficient to make him a very desirable match for Miss Darcy, and it was a great relief to Mrs. Bingley to be able to think of her sister as settled in the neighbourhood of Pemberley.

Jane and Elizabeth were now within a few miles of each other; and they had the pleasure of seeing each other almost every day. Mrs. Bennet was obliged to live alone, as her husband refused to leave Longbourn; but she was not unhappy without her family, as she had always been used to be alone. Mary was the only one of the family who was still at home; and she, though she could not like society, was by no means an enemy to it, and was gradually becoming less apt to shrink from it.

Mr. Collins was now married to Charlotte, and lived at Hunsford. Mr. Bennet had been to visit them there, and had been very much pleased with his reception. Mr. Collins was very attentive to his wife, and Mr. Bennet was glad to find that he was not unhappy with her. Mr. Collins was also very much pleased with his wife's relations, and was glad to be able to show them his attentions. Mr. Bennet was glad to find that his son-in-law was not a fool, and was glad to be able to visit him without being ashamed of him.

Mr. Bennet was also very much pleased with his visit to Pemberley. He was glad to find that his daughter was so happy, and was glad to be able to see her in her own home. He was also glad to find that his son-in-law was not a fool, and was glad to be able to visit him without being ashamed of him.

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Mr. Hall, of Vermont, moved that the Senate proceed to the consideration of executive business; which was agreed to.

After the consideration of executive business, the doors were opened, and the Senate adjourned.

THURSDAY, December 5.

Mr. Hall presented the petition of Jonathan Hunt, praying to be allowed a pension; which was referred to the Committee on Pensions

Lemma 2.1. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^n$ and $\mathcal{B}$ be a finite set of affine hyperplanes in $\mathbb{R}^m$. Then

$$\chi(\mathcal{A} \times \mathcal{B}, t) = \chi(\mathcal{A}, t)\chi(\mathcal{B}, t).$$

Proof. Let $\mathcal{A} = \{H_1, \ldots, H_k\}$ and $\mathcal{B} = \{H'_1, \ldots, H'_l\}$. Then $\mathcal{A} \times \mathcal{B} = \{H_1 \times \mathbb{R}^m, \ldots, H_k \times \mathbb{R}^m, \mathbb{R}^n \times H'_1, \ldots, \mathbb{R}^n \times H'_l\}$. Let R be a region of $\mathcal{A}$ and R' be a region of $\mathcal{B}$. Then $R \times R'$ is a region of $\mathcal{A} \times \mathcal{B}$. Conversely, every region of $\mathcal{A} \times \mathcal{B}$ is of this form. Thus $r(\mathcal{A} \times \mathcal{B}) = r(\mathcal{A})r(\mathcal{B})$. Similarly, $b(\mathcal{A} \times \mathcal{B}) = b(\mathcal{A})b(\mathcal{B})$. Hence

$$\chi(\mathcal{A} \times \mathcal{B}, t) = (-1)^{n+m} r(\mathcal{A} \times \mathcal{B}) - (-1)^{n+m-1} b(\mathcal{A} \times \mathcal{B})t$$
$$= (-1)^n r(\mathcal{A})(-1)^m r(\mathcal{B}) - (-1)^{n-1} b(\mathcal{A})t \cdot (-1)^{m-1} b(\mathcal{B})t$$
$$= \chi(\mathcal{A}, t)\chi(\mathcal{B}, t).$$

Lemma 2.2. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^n$ and $\mathcal{B}$ be a finite set of affine hyperplanes in $\mathbb{R}^m$. Then

$$\chi(\mathcal{A} \times \mathcal{B}, t) = \chi(\mathcal{A}, t)\chi(\mathcal{B}, t).$$

Proof. Let $\mathcal{A} = \{H_1, \ldots, H_k\}$ and $\mathcal{B} = \{H'_1, \ldots, H'_l\}$. Then $\mathcal{A} \times \mathcal{B} = \{H_1 \times \mathbb{R}^m, \ldots, H_k \times \mathbb{R}^m, \mathbb{R}^n \times H'_1, \ldots, \mathbb{R}^n \times H'_l\}$. Let R be a region of $\mathcal{A}$ and R' be a region of $\mathcal{B}$. Then $R \times R'$ is a region of $\mathcal{A} \times \mathcal{B}$. Conversely, every region of $\mathcal{A} \times \mathcal{B}$ is of this form. Thus $r(\mathcal{A} \times \mathcal{B}) = r(\mathcal{A})r(\mathcal{B})$. Similarly, $b(\mathcal{A} \times \mathcal{B}) = b(\mathcal{A})b(\mathcal{B})$. Hence

$$\chi(\mathcal{A} \times \mathcal{B}, t) = (-1)^{n+m} r(\mathcal{A} \times \mathcal{B}) - (-1)^{n+m-1} b(\mathcal{A} \times \mathcal{B})t$$
$$= (-1)^n r(\mathcal{A})(-1)^m r(\mathcal{B}) - (-1)^{n-1} b(\mathcal{A})t \cdot (-1)^{m-1} b(\mathcal{B})t$$
$$= \chi(\mathcal{A}, t)\chi(\mathcal{B}, t).$$

Dr. Hutton's Theory of the Earth.

Dr. Hutton's Theory of the Earth.

Dr. Hutton's Theory of the Earth.

Algorithm 1.1 Euclid's algorithm

Input: Two positive integers m and n.
Output: The greatest common divisor of m and n.

1. If $m < n$, swap m and n.
2. Divide m by n and let r be the remainder.
3. If $r = 0$, return n as the answer.
4. Replace m with n, replace n with r, and go to step 2.

Euclid's algorithm is based on the observation that the greatest common divisor of two numbers does not change if the smaller number is subtracted from the larger number. For example, the greatest common divisor of 60 and 24 is 12, and the greatest common divisor of 60 − 24 = 36 and 24 is also 12. Euclid's algorithm repeatedly applies this subtraction rule, but it uses the more efficient division instead of subtraction. Step 1 ensures that $m \geq n$, and step 2 replaces m with $m - qn$ for some integer q, which is the same as repeatedly subtracting n from m. Step 3 checks if the algorithm has finished, and step 4 replaces m with n and n with r to continue the algorithm.

Algorithm 1.2 Consecutive integer checking algorithm

Input: Two positive integers m and n.
Output: The greatest common divisor of m and n.

1. Let $t = \min(m, n)$.
2. If $m \bmod t = 0$ and $n \bmod t = 0$, return t as the answer.
3. Decrease t by 1 and go to step 2.

Algorithm 1.2 is another algorithm for finding the greatest common divisor of two numbers. It starts with the smaller number and checks if it is a divisor of both numbers. If it is, it returns the number as the answer. If it is not, it decreases the number by 1 and repeats the process. This algorithm is less efficient than Euclid's algorithm because it may take many iterations to find the greatest common divisor.

Algorithm 1.3 Middle-school algorithm

Input: Two positive integers m and n.
Output: The greatest common divisor of m and n.

1. Find the prime factorization of m.
2. Find the prime factorization of n.
3. Find all the common prime factors.
4. Compute the product of all the common prime factors and return it as the answer.

Algorithm 1.3 is the middle-school algorithm for finding the greatest common divisor of two numbers. It finds the prime factorization of both numbers, finds the common prime factors, and computes the product of the common prime factors. This algorithm is also less efficient than Euclid's algorithm because it requires finding the prime factorization of both numbers.



Abstract. This paper describes the use of the Springer LaTeX package, version 2.0. Springer Computer Science Proceedings uses Springer's standard document style "llncs" which is available from Springer's FTP server. The "llncs" style is a modification of the standard LaTeX article style and is used for Springer's Lecture Notes in Computer Science series and many Springer Computer Science Proceedings. The modifications include the following: a smaller text area, enabling authors to get about 45 lines per page; a different treatment of figure and table captions which allows for maximum clarity; redefinition of the thebibliography environment for a smaller font and closer spacing; redefinition of the itemize environment for more closely spaced lists; and numerous minor changes affecting only details. The "llncs" style will not necessarily be used by all Springer Computer Science Proceedings, so authors should look at the instructions for authors for the proceedings to which they are submitting papers.

Keywords: computer science, LaTeX, Springer-Verlag

1 Introduction

This paper describes the use of the Springer LaTeX package, version 2.0. Springer Computer Science Proceedings uses Springer's standard document style "llncs" which is available from Springer's FTP server. The "llncs" style is a modification of the standard LaTeX article style and is used for Springer's Lecture Notes in Computer Science series and many Springer Computer Science Proceedings. The modifications include the following: a smaller text area, enabling authors to get about 45 lines per page; a different treatment of figure and table captions which allows for maximum clarity; redefinition of the thebibliography environment for a smaller font and closer spacing; redefinition of the itemize environment for more closely spaced lists; and numerous minor changes affecting only details. The "llncs" style will not necessarily be used by all Springer Computer Science Proceedings, so authors should look at the instructions for authors for the proceedings to which they are submitting papers.

2 Typesetting Text

2.1 Headings

Headings should be capitalized (i.e., nouns, verbs, and all other words except articles, prepositions, and conjunctions should be set with an initial capital) and should, with the exception of the abstract, be numbered.

2.2 Lists

Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

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3 Typesetting Mathematics

3.1 Theorem-like Environments

Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

Theorem 1. Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

Proof. Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1. □

Definition 1. Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

3.2 Equations

Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

$$E = mc^2 \tag{1}$$

Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \tag{2}$$

4 Typesetting Figures and Tables

Figure 1. Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

Header 1	Header 2	Header 3
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Table 1. Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

5 Typesetting Bibliography

Lists may be numbered or bulleted, and may be lettered or numbered consecutively within a section or with each section starting over again with number 1.

[1] Author, A.: Title of the paper. Journal Name, Vol. 1, No. 1, pp. 1–10 (2000)
[2] Author, B.: Title of the book. Publisher, City (2001)

[illegible] [illegible] [illegible] [

Alexander's mother, Olympias, was a princess of Epirus, and was
married to Philip of Macedon, who was then a young man. Olympias
was a very beautiful woman, but she was also very fierce and
passionate. Philip was a great warrior, and he was away from home
a great deal of the time, fighting battles and conquering countries.
Olympias was very jealous of him, and she was very unhappy when he
was away. She used to shut herself up in her room and weep and
wail, and refuse to see anybody. Sometimes she would even refuse to
eat, and would starve herself for days together.

Philip was very fond of Olympias, and he was very sorry to see her
unhappy. He used to try to comfort her when he came home, and he
would bring her presents of jewels and fine clothes. But she would
not be comforted, and she would still shut herself up in her room
and weep and wail.

One day Philip came home from a battle, and he found Olympias in
her room, weeping and wailing as usual. He went to her and tried to
comfort her, but she would not be comforted. She said that she was
sure that he would be killed in battle, and that she would never see
him again.

Philip was very angry with her for saying this. He told her that
she was a foolish woman, and that she must not talk like that. He
said that he was a great warrior, and that he was not afraid of
death. He said that he would come home safe and sound, and that she
must not weep and wail any more.

Olympias was very much frightened by Philip's anger. She stopped
weeping and wailing, and she promised that she would not talk like
that any more. Philip was very glad to see her stop weeping and
wailing, and he was very glad to hear her promise that she would not
talk like that any more.

Philip went away to fight another battle, and Olympias was very
unhappy when he was away. She shut herself up in her room and wept
and wailed, and refused to see anybody. She refused to eat, and she
starved herself for days together.

Philip came home from the battle, and he found Olympias in her

Each of the following is a statement of the Pythagorean theorem:

(1) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

(2) In a right triangle, the sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse.

(3) In a right triangle, the sum of the areas of the similar figures on the legs is equal to the area of the similar figure on the hypotenuse.

(4) In a right triangle, the sum of the areas of the similar isosceles triangles on the legs is equal to the area of the similar isosceles triangle on the hypotenuse.

(5) In a right triangle, the sum of the areas of the similar semicircles on the legs is equal to the area of the similar semicircle on the hypotenuse.

(6) In a right triangle, the sum of the areas of the similar regular hexagons on the legs is equal to the area of the similar regular hexagon on the hypotenuse.

(7) In a right triangle, the sum of the areas of the similar regular pentagons on the legs is equal to the area of the similar regular pentagon on the hypotenuse.

(8) In a right triangle, the sum of the areas of the similar regular octagons on the legs is equal to the area of the similar regular octagon on the hypotenuse.

(9) In a right triangle, the sum of the areas of the similar regular n-gons on the legs is equal to the area of the similar regular n-gon on the hypotenuse.

(10) In a right triangle, the sum of the areas of the similar ellipses on the legs is equal to the area of the similar ellipse on the hypotenuse.

(11) In a right triangle, the sum of the areas of the similar parallelograms on the legs is equal to the area of the similar parallelogram on the hypotenuse.

(12) In a right triangle, the sum of the areas of the similar rectangles on the legs is equal to the area of the similar rectangle on the hypotenuse.

(13) In a right triangle, the sum of the areas of the similar rhombuses on the legs is equal to the area of the similar rhombus on the hypotenuse.

(14) In a right triangle, the sum of the areas of the similar trapezoids on the legs is equal to the area of the similar trapezoid on the hypotenuse.

(15) In a right triangle, the sum of the areas of the similar triangles on the legs is equal to the area of the similar triangle on the hypotenuse.

(16) In a right triangle, the sum of the areas of the similar quadrilaterals on the legs is equal to the area of the similar quadrilateral on the hypotenuse.

(17) In a right triangle, the sum of the areas of the similar polygons on the legs is equal to the area of the similar polygon on the hypotenuse.

(18) In a right triangle, the sum of the areas of the similar figures on the legs is equal to the area of the similar figure on the hypotenuse.

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Mississippi River, and the Iowa River, which flows into the Mississippi from the west, and the Des Moines River, which flows into the Mississippi from the northwest, and the Missouri River, which flows into the Mississippi from the west, and the Platte River, which flows into the Missouri from the west, and the Kansas River, which flows into the Missouri from the west, and the Arkansas River, which flows into the Mississippi from the west, and the Red River, which flows into the Mississippi from the west, and the Ohio River, which flows into the Mississippi from the east, and the Illinois River, which flows into the Mississippi from the east, and the Wisconsin River, which flows into the Mississippi from the east, and the Minnesota River, which flows into the Mississippi from the northwest, and the St. Croix River, which flows into the Mississippi from the northwest, and the Chippewa River, which flows into the Mississippi from the northwest, and the Black River, which flows into the Mississippi from the northwest, and the Root River, which flows into the Mississippi from the northwest, and the Upper Iowa River, which flows into the Mississippi from the northwest, and the Turkey River, which flows into the Mississippi from the northwest, and the Maquoketa River, which flows into the Mississippi from the northwest, and the Wapsipinicon River, which flows into the Mississippi from the northwest, and the Cedar River, which flows into the Mississippi from the northwest, and the Iowa River, which flows into the Mississippi from the northwest, and the Des Moines River, which flows into the Mississippi from the northwest, and the Missouri River, which flows into the Mississippi from the northwest, and the Platte River, which flows into the Missouri from the northwest, and the Kansas River, which flows into the Missouri from the northwest, and the Arkansas River, which flows into the Mississippi from the northwest, and the Red River, which flows into the Mississippi from 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which flows into the Mississippi from the southeast, and the Ohio River, which flows into the Mississippi from the southwest, and the Illinois River, which flows into the Mississippi from the southwest, and the Wisconsin River, which flows into the Mississippi from the southwest, and the Minnesota River, which flows into the Mississippi from the southwest, and the St. Croix River, which flows into the Mississippi from the southwest, and the Chippewa River, which flows into the Mississippi from the southwest, and the Black River, which flows into the Mississippi from the southwest, and the Root River, which flows into the Mississippi from the southwest, and the Upper Iowa River, which flows into the Mississippi from the southwest, and the Turkey River, which flows into the Mississippi from the southwest, and the Maquoketa River, which flows into the Mississippi from the southwest, and the Wapsipinicon River, which flows into the Mississippi from the southwest, and the Cedar River, which 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Suppose you are given a list of 1,000 numbers and asked to find the largest number in the list. You would probably look at the first number, then look at the second number and remember it if it is larger, then look at the third number and remember it if it is larger, and so on. This algorithm for finding the largest number in a list requires 1,000 steps. Now suppose you are given a list of 1,000 numbers and asked to find the 100 largest numbers in the list. You could repeat the algorithm for finding the largest number 100 times, but this would require 100,000 steps. Now suppose you are given a list of 1,000 numbers and asked to find the median number in the list. You could sort the list and then look at the middle number, but sorting the list would require more than 1,000 steps. Now suppose you are given a list of 1,000 numbers and asked to find the 100 numbers closest to the median number in the list. You could find the median number and then look at the 100 numbers on either side of it, but finding the median number would require more than 1,000 steps.

These examples illustrate the importance of choosing the right algorithm for the job. The first example required 1,000 steps, the second example required 100,000 steps, and the third example required more than 1,000 steps. The fourth example required more than 1,000 steps plus the number of steps required to find the median number. The number of steps required by an algorithm is called its *time complexity*. The time complexity of an algorithm is usually expressed as a function of the size of the input. For example, the time complexity of the algorithm for finding the largest number in a list of 1,000 numbers is 1,000 steps. The time complexity of the algorithm for finding the 100 largest numbers in a list of 1,000 numbers is 100,000 steps. The time complexity of the algorithm for finding the median number in a list of 1,000 numbers is more than 1,000 steps. The time complexity of the algorithm for finding the 100 numbers closest to the median number in a list of 1,000 numbers is more than 1,000 steps plus the number of steps required to find the median number.

The time complexity of an algorithm is usually expressed using *big O notation*. Big O notation is a way of describing the upper bound of the time complexity of an algorithm. For example, the time complexity of the algorithm for finding the largest number in a list of 1,000 numbers is $O(1000)$. The time complexity of the algorithm for finding the 100 largest numbers in a list of 1,000 numbers is $O(100000)$. The time complexity of the algorithm for finding the median number in a list of 1,000 numbers is $O(n)$, where n is the number of numbers in the list. The time complexity of the algorithm for finding the 100 numbers closest to the median number in a list of 1,000 numbers is $O(n)$, where n is the number of numbers in the list.

Big O notation is useful because it allows us to compare the time complexity of different algorithms. For example, we can say that the algorithm for finding the largest number in a list of 1,000 numbers is faster than the algorithm for finding the 100 largest numbers in a list of 1,000 numbers because $O(1000)$ is less than $O(100000)$. We can also say that the algorithm for finding the median number in a list of 1,000 numbers is faster than the algorithm for finding the 100 numbers closest to the median number in a list of 1,000 numbers because $O(n)$ is less than $O(n)$ plus the number of steps required to find the median number.

Big O notation is also useful because it allows us to predict the time complexity of an algorithm for a given input size. For example, we can predict that the algorithm for finding the largest number in a list of 1,000 numbers will take 1,000 steps. We can also predict that the algorithm for finding the 100 largest numbers in a list of 1,000 numbers will take 100,000 steps. We can also predict that the algorithm for finding the median number in a list of 1,000 numbers will take more than 1,000 steps. We can also predict that the algorithm for finding the 100 numbers closest to the median number in a list of 1,000 numbers will take more than 1,000 steps plus the number of steps required to find the median number.

Big O notation is also useful because it allows us to compare the time complexity of an algorithm for different input sizes. For example, we can say that the algorithm for finding the largest number in a list of 1,000 numbers is faster than the algorithm for finding the largest number in a list of 10,000 numbers because $O(1000)$ is less than $O(10000)$. We can also say that the algorithm for finding the median number in a list of 1,000 numbers is faster than the algorithm for finding the median number in a list of 10,000 numbers because $O(n)$ is less than $O(n)$ plus the number of steps required to find the median number.

Big O notation is also useful because it allows us to compare the time complexity of an algorithm for different input sizes. For example, we can say that the algorithm for finding the largest number in a list of 1,000 numbers is faster than the algorithm for finding the largest number in a list of 10,000 numbers because $O(1000)$ is less than $O(10000)$. We can also say that the algorithm for finding the median number in a list of 1,000 numbers is faster than the algorithm for finding the median number in a list of 10,000 numbers because $O(n)$ is less than $O(n)$ plus the number of steps required to find the median number.

Christ, and the Church, which is His Body, is the Bride. Christ is the Head of the Church, and the Church is subject to Christ. Christ is the Vine, and the Church is the branches. Christ is the Foundation, and the Church is the building. Christ is the Shepherd, and the Church is the sheep. Christ is the Bridegroom, and the Church is the Bride. Christ is the Husband, and the Church is the wife. Christ is the Lord, and the Church is the servant. Christ is the Master, and the Church is the disciple. Christ is the Teacher, and the Church is the learner. Christ is the Leader, and the Church is the follower. Christ is the Captain, and the Church is the soldier. Christ is the King, and the Church is the subject. Christ is the Priest, and the Church is the people. Christ is the Prophet, and the Church is the hearer. Christ is the Judge, and the Church is the witness. Christ is the Redeemer, and the Church is the redeemed. Christ is the Saviour, and the Church is the saved. Christ is the Deliverer, and the Church is the delivered. Christ is the Healer, and the Church is the healed. Christ is the Comforter, and the Church is the comforted. Christ is the Helper, and the Church is the helped. Christ is the Guide, and the Church is the guided. Christ is the Protector, and the Church is the protected. Christ is the Provider, and the Church is the provided for. Christ is the Sustainer, and the Church is the sustained. Christ is the Keeper, and the Church is the kept. Christ is the Preserver, and the Church is the preserved. Christ is the Perfecter, and the Church is the perfected. Christ is the Finisher, and the Church is the finished. Christ is the Author, and the Church is the work. Christ is the Beginning, and the Church is the end. Christ is the Alpha, and the Church is the Omega. Christ is the First, and the Church is the last. Christ is the Eternal, and the Church is the temporal. Christ is the Infinite, and the Church is the finite. Christ is the Creator, and the Church is the created. Christ is the Maker, and the Church is the made. Christ is the Builder, and the Church is the built. Christ is the Architect, and the Church is the plan. Christ is the Designer, and the Church is the design. Christ is the Artist, and the Church is the picture. Christ is the Sculptor, and the Church is the statue. Christ is the Musician, and the Church is the song. Christ is the Poet, and the Church is the poem. Christ is the Author, and the Church is the book. Christ is the Writer, and the Church is the letter. Christ is the Speaker, and the Church is the word. Christ is the Voice, and the Church is the echo. Christ is the Light, and the Church is the reflection. Christ is the Sun, and the Church is the moon. Christ is the Star, and the Church is the planet. Christ is the Fire, and the Church is the heat. Christ is the Water, and the Church is the wave. Christ is the Air, and the Church is the breath. Christ is the Earth, and the Church is the soil. Christ is the Rock, and the Church is the stone. Christ is the Tree, and the Church is the fruit. Christ is the Seed, and the Church is the harvest. Christ is the Sower, and the Church is the reaper. Christ is the Gardener, and the Church is the garden. Christ is the Husbandman, and the Church is the vineyard. Christ is the Shepherd, and the Church is the flock. Christ is the Fisherman, and the Church is the net. Christ is the Hunter, and the Church is the prey. Christ is the Warrior, and the Church is the battle. Christ is the Conqueror, and the Church is the victory. Christ is the Champion, and the Church is the crown. Christ is the Athlete, and the Church is the race. Christ is the Runner, and the Church is the goal. Christ is the Pilgrim, and the Church is the journey. Christ is the Traveler, and the Church is the road. Christ is the Way, and the Church is the walk. Christ is the Path, and the Church is the step. Christ is the Door, and the Church is the entrance. Christ is the Gate, and the Church is the passage. Christ is the Bridge, and the Church is the crossing. Christ is the Ladder, and the Church is the climb. Christ is the Stairway, and the Church is the ascent. Christ is the Elevator, and the Church is the lift. Christ is the Escalator, and the Church is the rise. Christ is the Mountain, and the Church is the peak. Christ is the Valley, and the Church is the depth. Christ is the Ocean, and the Church is the shore. Christ is the River, and the Church is the stream. Christ is the Lake, and the Church is the pool. Christ is the Spring, and the Church is the well. Christ is the Fountain, and the Church is the drink. Christ is the Bread, and the Church is the feast. Christ is the Wine, and the Church is the cup. Christ is the Meat, and the Church is the table. Christ is the Host, and the Church is the guest. Christ is the Master, and the Church is the servant. Christ is the Lord, and the Church is the slave. Christ is the King, and the Church is the subject. Christ is the Father, and the Church is the child. Christ is the Mother, and the Church is the nurse. Christ is the Brother, and the Church is the sister. Christ is the Friend, and the Church is the friend. Christ is the Neighbor, and the Church is the neighbor. Christ is the Stranger, and the Church is the stranger. Christ is the Enemy, and the Church is the enemy. Christ is the Judge, and the Church is the criminal. Christ is the Lawyer, and the Church is the client. Christ is the Doctor, and the Church is the patient. Christ is the Nurse, and the Church is the sick. Christ is the Teacher, and the Church is the student. Christ is the Professor, and the Church is the pupil. Christ is the Scholar, and the Church is the learner. Christ is the Master, and the Church is the apprentice. Christ is the Craftsman, and the Church is the tool. Christ is the Builder, and the Church is the material. Christ is the Architect, and the Church is the blueprint. Christ is the Engineer, and the Church is the machine. Christ is the Mechanic, and the Church is the repair. Christ is the Farmer, and the Church is the crop. Christ is the Rancher, and the Church is the cattle. Christ is the Miner, and the Church is the ore. Christ is the Fisherman, and the Church is the catch. Christ is the Hunter, and the Church is the game. Christ is the Trapper, and the Church is the fur. Christ is the Logger, and the Church is the timber. Christ is the Sailor, and the Church is the ship. Christ is the Pilot, and the Church is the navigation. Christ is the Captain, and the Church is the crew. Christ is the Soldier, and the Church is the army. Christ is the General, and the Church is the troops. Christ is the Admiral, and the Church is the fleet. Christ is the Pilot, and the Church is the plane. Christ is the Driver, and the Church is the car. Christ is the Engineer, and the Church is the train. Christ is the Conductor, and the Church is the passenger. Christ is the Merchant, and the Church is the goods. Christ is the Banker, and the Church is the money. Christ is the Trader, and the Church is the exchange. Christ is the Buyer, and the Church is the sale. Christ is the Seller, and the Church is the purchase. Christ is the Customer, and the Church is the service. Christ is the Waiter, and the Church is the meal. Christ is the Cook, and the Church is the food. Christ is the Baker, and the Church is the bread. Christ is the Butcher, and the Church is the meat. Christ is the Grocer, and the Church is the supply. Christ is the Tailor, and the Church is the clothing. Christ is the Shoemaker, and the Church is the shoe. Christ is the Hatter, and the Church is the hat. Christ is the Jeweler, and the Church is the gem. Christ is the Goldsmith, and the Church is the gold. Christ is the Silversmith, and the Church is the silver. Christ is the Blacksmith, and the Church is the iron. Christ is the Carpenter, and the Church is the wood. Christ is the Mason, and the Church is the stone. Christ is the Painter, and the Church is the color. Christ is the Glazier, and the Church is the glass. Christ is the Roofer, and the Church is the shelter. Christ is the Plumber, and the Church is the water. Christ is the Electrician, and the Church is the light. Christ is the Gardener, and the Church is the flower. Christ is the Florist, and the Church is the bouquet. Christ is the Barber, and the Church is the hair. Christ is the Doctor, and the Church is the health. Christ is the Dentist, and the Church is the tooth. Christ is the Optician, and the Church is the vision. Christ is the Veterinarian, and the Church is the animal. Christ is the Pharmacist, and the Church is the medicine. Christ is the Nurse, and the Church is the care. Christ is the Midwife, and the Church is the birth. Christ is the Undertaker, and the Church is the burial. Christ is the Minister, and the Church is the congregation. Christ is the Priest, and the Church is the altar. Christ is the Rabbi, and the Church is the synagogue. Christ is the Imam, and the Church is the mosque. Christ is the Monk, and the Church is the monastery. Christ is the Nun, and the Church is the convent. Christ is the Pope, and the Church is the Vatican. Christ is the Bishop, and the Church is the diocese. Christ is the Archbishop, and the Church is the province. Christ is the Cardinal, and the Church is the college. Christ is the Deacon, and the Church is the service. Christ is the Elder, and the Church is the session. Christ is the Pastor, and the Church is the flock. Christ is the Evangelist, and the Church is the gospel. Christ is the Missionary, and the Church is the mission. Christ is the Apostle, and the Church is the message. Christ is the Prophet, and the Church is the vision. Christ is the Seer, and the Church is the revelation. Christ is the Teacher, and the Church is the doctrine. Christ is the Preacher, and the Church is the sermon. Christ is the Evangelist, and the Church is the invitation. Christ is the Counselor, and the Church is the advice. Christ is the Comforter, and the Church is the consolation. Christ is the Intercessor, and the Church is the prayer. Christ is the Mediator, and the Church is the reconciliation. Christ is the Advocate, and the Church is the defense. Christ is the Witness, and the Church is the testimony. Christ is the Martyr, and the Church is the faith. Christ is the Saint, and the Church is the holiness. Christ is the Sinner, and the Church is the repentance. Christ is the Penitent, and the Church is the confession. Christ is the Forgiver, and the Church is the pardon. Christ is the Redeemer, and the Church is the ransom. Christ is the Savior, and the Church is the salvation. Christ is the Deliverer, and the Church is the freedom. Christ is the Liberator, and the Church is the liberty. Christ is the Emancipator, and the Church is the slave. Christ is the Abolitionist, and the Church is the abolition. Christ is the Reformer, and the Church is the reform. Christ is the Revolutionary, and the Church is the revolution. Christ is the Rebel, and the Church is the rebellion. Christ is the Traitor, and the Church is the betrayal. Christ is the Spy, and the Church is the espionage. Christ is the Agent, and the Church is the mission. Christ is the Ambassador, and the Church is the embassy. Christ is the Diplomat, and the Church is the treaty. Christ is the Politician, and the Church is the party. Christ is the Statesman, and the Church is the nation. Christ is the President, and the Church is the republic. Christ is the Governor, and the Church is the state. Christ is the Mayor, and the Church is the city. Christ is the Sheriff, and the Church is the county. Christ is the Judge, and the Church is the court. Christ is the Jury, and the Church is the verdict. Christ is the Lawyer, and the Church is the case. Christ is the Client, and the Church is the cause. Christ is the Witness, and the Church is the testimony. Christ is the Defendant, and the Church is the plaintiff. Christ is the Prisoner, and the Church is the jail. Christ is the Warden, and the Church is the prison. Christ is the Guard, and the Church is the watch. Christ is the Soldier, and the Church is the army. Christ is the General, and the Church is the troops. Christ is the Admiral, and the Church is the fleet. Christ is the Pilot, and the Church is the plane. Christ is the Driver, and the Church is the car. Christ is the Engineer, and the Church is the train. Christ is the Conductor, and the Church is the passenger. Christ is the Merchant, and the Church is the goods. Christ is the Banker, and the Church is the money. Christ is the Trader, and the Church is the exchange. Christ is the Buyer, and the Church is the sale. Christ is the Seller, and the Church is the purchase. Christ is the Customer, and the Church is the service. Christ is the Waiter, and the Church is the meal. Christ is the Cook, and the Church is the food. Christ is the Baker, and the Church is the bread. Christ is the Butcher, and the Church is the meat. Christ is the Grocer, and the Church is the supply. Christ is the Tailor, and the Church is the clothing. Christ is the Shoemaker, and the Church is the shoe. Christ is the Hatter, and the Church is the hat. Christ is the Jeweler, and the Church is the gem. Christ is the Goldsmith, and the Church is the gold. Christ is the Silversmith, and the Church is the silver. Christ is the Blacksmith, and the Church is the iron. Christ is the Carpenter, and the Church is the wood. Christ is the Mason, and the Church is the stone. Christ is the Painter, and the Church is the color. Christ is the Glazier, and the Church is the glass. Christ is the Roofer, and the Church is the shelter. Christ is the Plumber, and the Church is the water. Christ is the Electrician, and the Church is the light. Christ is the Gardener, and the Church is the flower. Christ is the Florist, and the Church is the bouquet. Christ is the Barber, and the Church is the hair. Christ is the Doctor, and the Church is the health. Christ is the Dentist, and the Church is the tooth. Christ is the Optician, and the Church is the vision. Christ is the Veterinarian, and the Church is the animal. Christ is the Pharmacist, and the Church is the medicine. Christ is the Nurse, and the Church is the care. Christ is the Midwife, and the Church is the birth. Christ is the Undertaker, and the Church is the burial. Christ is the Minister, and the Church is the congregation. Christ is the Priest, and the Church is the altar. Christ is the Rabbi, and the Church is the synagogue. Christ is the Imam, and the Church is the mosque. Christ is the Monk, and the Church is the monastery. Christ is the Nun, and the Church is the convent. Christ is the Pope, and the Church is the Vatican. Christ is the Bishop, and the Church is the diocese. Christ is the Archbishop, and the Church is the province. Christ is the Cardinal, and the Church is the college. Christ is the Deacon, and the Church is the service. Christ is the Elder, and the Church is the session. Christ is the Pastor, and the Church is the flock. Christ is the Evangelist, and the Church is the gospel. Christ is the Missionary, and the Church is the mission. Christ is the Apostle, and the Church is the message. Christ is the Prophet, and the Church is the vision. Christ is the Seer, and the Church is the revelation. Christ is the Teacher, and the Church is the doctrine. Christ is the Preacher, and the Church is the sermon. Christ is the Evangelist, and the Church is the invitation. Christ is the Counselor, and the Church is the advice. Christ is the Comforter, and the Church is the consolation. Christ is the Intercessor, and the Church is the prayer. Christ is the Mediator, and the Church is the reconciliation. Christ is the Advocate, and the Church is the defense. Christ is the Witness, and the Church is the testimony. Christ is the Martyr, and the Church is the faith. Christ is the Saint, and the Church is the holiness. Christ is the Sinner, and the Church is the repentance. Christ is the Penitent, and the Church is the confession. Christ is the Forgiver, and the Church is the pardon. Christ is the Redeemer, and the Church is the ransom. Christ is the Savior, and the Church is the salvation. Christ is the Deliverer, and the Church is the freedom. Christ is the Liberator, and the Church is the liberty. Christ is the Emancipator, and the Church is the slave. Christ is the Abolitionist, and the Church is the abolition. Christ is the Reformer, and the Church is the reform. Christ is the Revolutionary, and the Church is the revolution. Christ is the Rebel, and the Church is the rebellion. Christ is the Traitor, and the Church is the betrayal. Christ is the Spy, and the Church is the espionage. Christ is the Agent, and the Church is the mission. Christ is the Ambassador, and the Church is the embassy. Christ is the Diplomat, and the Church is the treaty. Christ is the Politician, and the Church is the party. Christ is the Statesman, and the Church is the nation. Christ is the President, and the Church is the republic. Christ is the Governor, and the Church is the state. Christ is the Mayor, and the Church is the city. Christ is the Sheriff, and the Church is the county. Christ is the Judge, and the Church is the court. Christ is the Jury, and the Church is the verdict. Christ is the Lawyer, and the Church is the case. Christ is the Client, and the Church is the cause. Christ is the Witness, and the Church is the testimony. Christ is the Defendant, and the Church is the plaintiff. Christ is the Prisoner, and the Church is the jail. Christ is the Warden, and the Church is the prison. Christ is the Guard, and the Church is the watch.

Figure 1.1 shows the relationship between the problem, the algorithm, and the program. A problem can be solved by many algorithms. An algorithm can be implemented by many programs, depending on the programming language and the programmer's style. Figure 1.1 also shows that we can analyze algorithms and programs, and that we can test programs.



Figure 1.1 The problem, the algorithm, and the program

Algorithm analysis is the process of determining the amount of computer time and storage that algorithms will require. Program testing is the process of executing a program to show that it works correctly. Program testing can be used to show the presence of bugs, but rarely to show their absence. Algorithm analysis is a theoretical process that can be carried out before the program is written.

Algorithm analysis is important because it can help us to understand the scalability of our programs and the reasons why some programs require more resources than their functionality warrants. Moreover, algorithm analysis is frequently an interesting and challenging activity in its own right.

Algorithm analysis is sometimes called algorithmics. Algorithmics is more than the implementation, testing, and analysis of algorithms. Algorithmics is the overall study of algorithms. It includes activities such as creating algorithms, proving algorithms correct, studying the inherent complexity of problems, and investigating algorithms that manipulate abstract mathematical objects.

Algorithmics is a major theoretical part of computer science. Theoretical computer science is important to practicing programmers because it provides the fundamental knowledge of what is computable and how efficiently it can be computed. Theoretical computer science is fun because it is challenging and accessible. Anyone who can write a computer program can participate in the discovery of new algorithms and the analysis of algorithms.

1.2 Data Structures

Data structures are the basic building blocks of algorithms. Algorithms are the basic building blocks of programs. Therefore, data structures are the basic building blocks of programs. Data structures are the subject of this book.

Data structures are the ways that algorithms see data. Algorithms are the ways that programs see data. Therefore, data structures are the ways that programs see data. Data structures are the ways that programs see the world

Cicero's letters to Atticus are full of references to the cost of living in Rome. Atticus was a wealthy man, but he was also a careful one, and he kept a close eye on his expenses. Cicero, who was always in debt, envied his friend's financial discipline. In one letter he complains that he cannot afford to entertain as he would like, because the price of food has risen so sharply that a dinner party for twelve guests would cost him more than he could comfortably spare.

The rise in prices was not confined to luxuries. Even the basic grain ration, which the state distributed to the poor, was subject to fluctuations that could cause real hardship. When the harvest failed in Africa or Sicily, the consequences were felt within weeks in the streets of Rome. Cicero himself, as consul, had to deal with a shortage in 57 BC, and he recalled the experience with evident relief that it had not become a crisis.

Example 1 Identifying Subsets of the Real Numbers

Let $A = \{-\frac{1}{2}, 0, \frac{1}{3}, 0.75, 1, \sqrt{2}, \sqrt{4}, \sqrt{5}, \pi\}$. List the elements of A that are

a. natural numbers
b. whole numbers
c. integers
d. rational numbers
e. irrational numbers
f. real numbers

Solution:

a. The natural numbers are the counting numbers: $1, 2, 3, \ldots$. Thus, the natural numbers in A are 1 and $\sqrt{4}$ ($\sqrt{4} = 2$).

b. The whole numbers are the natural numbers and zero. Thus, the whole numbers in A are 0, 1, and $\sqrt{4}$.

c. The integers are the whole numbers and their opposites. Thus, the integers in A are 0, 1, $\sqrt{4}$, and $-\frac{1}{2}$.

d. The rational numbers are numbers that can be written as the ratio of two integers. Thus, the rational numbers in A are $-\frac{1}{2}$, 0, $\frac{1}{3}$, 0.75, 1, and $\sqrt{4}$.

e. The irrational numbers are numbers that cannot be written as the ratio of two integers. Thus, the irrational numbers in A are $\sqrt{2}$, $\sqrt{5}$, and π.

f. The real numbers are all the numbers in A.

Skill Practice 1

Let $B = \{-\frac{2}{3}, 0, \frac{1}{4}, 0.5, 1, \sqrt{3}, \sqrt{9}, \sqrt{10}, \pi\}$. List the elements of B that are

a. natural numbers
b. whole numbers
c. integers
d. rational numbers
e. irrational numbers
f. real numbers

Example 2 Identifying Subsets of the Real Numbers

Let $C = \{-\sqrt{3}, -\frac{5}{2}, -1, -0.2, 0, \frac{1}{2}, 1, \sqrt{7}, \sqrt{9}\}$. List the elements of C that are

a. natural numbers
b. whole numbers
c. integers
d. rational numbers
e. irrational numbers
f. real numbers

Solution:

a. The natural numbers are the counting numbers: $1, 2, 3, \ldots$. Thus, the natural numbers in C are 1 and $\sqrt{9}$ ($\sqrt{9} = 3$).

b. The whole numbers are the natural numbers and zero. Thus, the whole numbers in C are 0, 1, and $\sqrt{9}$.

c. The integers are the whole numbers and their opposites. Thus, the integers in C are -1, 0, 1, and $\sqrt{9}$.

d. The rational numbers are numbers that can be written as the ratio of two integers. Thus, the rational numbers in C are $-\frac{5}{2}$, -1, -0.2, 0, $\frac{1}{2}$, 1, and $\sqrt{9}$.

e. The irrational numbers are numbers that cannot be written as the ratio of two integers. Thus, the irrational numbers in C are $-\sqrt{3}$ and $\sqrt{7}$.

f. The real numbers are all the numbers in C.

Skill Practice 2

Let $D = \{-\sqrt{5}, -\frac{3}{4}, -1, -0.1, 0, \frac{1}{3}, 1, \sqrt{6}, \sqrt{16}\}$. List the elements of D that are

a. natural numbers
b. whole numbers
c. integers
d. rational numbers
e. irrational numbers
f. real numbers

Example 3 Identifying Subsets of the Real Numbers

Let $E = \{-\frac{1}{3}, 0, \frac{1}{2}, 0.25, 1, \sqrt{3}, \sqrt{4}, \sqrt{7}, \pi\}$. List the elements of E that are

a. natural numbers
b. whole numbers
c. integers
d. rational numbers
e. irrational numbers
f. real numbers

Solution:

a. The natural numbers are the counting numbers: $1, 2, 3, \ldots$. Thus, the natural numbers in E are 1 and $\sqrt{4}$ ($\sqrt{4} = 2$).

b. The whole numbers are the natural numbers and zero. Thus, the whole numbers in E are 0, 1, and $\sqrt{4}$.

c. The integers are the whole numbers and their opposites. Thus, the integers in E are 0, 1, $\sqrt{4}$, and $-\frac{1}{3}$.

d. The rational numbers are numbers that can be written as the ratio of two integers. Thus, the rational numbers in E are $-\frac{1}{3}$, 0, $\frac{1}{2}$, 0.25, 1, and $\sqrt{4}$.

e. The irrational numbers are numbers that cannot be written as the ratio of two integers. Thus, the irrational numbers in E are $\sqrt{3}$, $\sqrt{7}$, and π.

f. The real numbers are all the numbers in E.

Skill Practice 3

Let $F = \{-\frac{2}{5}, 0, \frac{1}{3}, 0.75, 1, \sqrt{2}, \sqrt{4}, \sqrt{5}, \pi\}$. List the elements of F that are

a. natural numbers
b. whole numbers
c. integers
d. rational numbers
e. irrational numbers
f. real numbers

Example 4 Identifying Subsets of the Real Numbers

Let $G = \{-\sqrt{2}, -\frac{3}{2}, -1, -0.3, 0, \frac{1}{3}, 1, \sqrt{5}, \sqrt{9}\}$. List the elements of G that are

a. natural numbers
b. whole numbers
c. integers
d. rational numbers
e. irrational numbers
f. real numbers

Solution:

a. The natural numbers are the counting numbers: $1, 2, 3, \ldots$. Thus, the natural numbers in G are 1 and $\sqrt{9}$ ($\sqrt{9} = 3$).

b. The whole numbers are the natural numbers and zero. Thus, the whole numbers in G are 0, 1, and $\sqrt{9}$.

c. The integers are the whole numbers and their opposites. Thus, the integers in G are -1, 0, 1, and $\sqrt{9}$.

d. The rational numbers are numbers that can be written as the ratio of two integers. Thus, the rational numbers in G are $-\frac{3}{2}$, -1, -0.3, 0, $\frac{1}{3}$, 1, and $\sqrt{9}$.

e. The irrational numbers are numbers that cannot be written as the ratio of two integers. Thus, the irrational numbers in G are $-\sqrt{2}$ and $\sqrt{5}$.

f. The real numbers are all the numbers in G.

Skill Practice 4

Let $H = \{-\sqrt{7}, -\frac{5}{4}, -1, -0.4, 0, \frac{1}{5}, 1, \sqrt{8}, \sqrt{16}\}$. List the elements of H that are

a. natural numbers
b. whole numbers
c. integers
d. rational numbers
e. irrational numbers
f. real numbers

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Ezra 1 1 Now in the first year of Cyrus king of the Persians, that the word of the Lord might be fulfilled which he spoke by the mouth of Jeremiah, 2 the Lord stirred up the spirit of Cyrus king of the Persians, and he made a proclamation throughout all his kingdom, and also by writing, saying, 3 Thus says Cyrus king of the Persians: The Lord God of heaven has made me king of the world, 4 and he has commanded me to build him a house in Jerusalem which is in Judea. If any of you are among his people, may his God be with him, and let him go up to Jerusalem which is in Judea, and let him build the house of the Lord God of Israel, the God who is in Jerusalem. 5 And let every survivor, in whatever place he sojourns, be helped by the men of his place with silver and gold, with goods and with cattle, besides the votive offerings for the house of God which is in Jerusalem. 6 Then the heads of families of Judah and Benjamin, and the priests and Levites, and all those whose spirit the Lord had stirred up, arose to go up to build the house of the Lord which is in Jerusalem. 7 And all their neighbors helped them with articles of silver and gold, with goods and with cattle, and with many votive offerings, whose minds were stirred up. 8 Cyrus the king also brought out the vessels of the house of the Lord, which Nebuchadnezzar had carried away from Jerusalem and had placed in the house of his god. 9 Cyrus king of the Persians brought them out by the hand of Mithridates the treasurer, and numbered them to Sanabassar, the governor of Judea. 10 And this was the number of them: thirty golden cups, one thousand silver cups, twenty-nine knives, 11 thirty golden bowls, two thousand four hundred and ten silver bowls, one thousand other vessels. 12 All the vessels of gold and of silver that were carried away at that time were five thousand four hundred and sixty-nine. 13 These were brought back by Sanabassar, when he came up from the captivity of those who were transported from Babylon to Jerusalem.

2 1 Now these are the children of the province who came up from the captivity of those who were transported, whom Nebuchadnezzar king of Babylon had carried away to Babylon, and who returned to Jerusalem and Judea, each to his own city, 2 who came with Zerubbabel, Jeshua, Nehemiah, Seraiah, Reelaiah, Mordecai, Bilshan, Mispar, Bigvai, Rehum, and Baanah. The number of the men of the people: 3 the children of Parosh, two thousand one hundred and seventy-two; 4 the children of Shephatiah, three hundred and seventy-two; 5 the children of Arah, seven hundred and seventy-five; 6 the children of Pahath-moab, of the children of Jeshua and Joab, two thousand eight hundred and twelve; 7 the children of Elam, one thousand two hundred and fifty-four; 8 the children of Zattu, nine hundred and forty-five; 9 the children of Chorbe, seven hundred and sixty; 10 the children of Bebai, six hundred and forty-three; 11 the children of Azgad, one thousand three hundred and twenty-two; 12 the children of Adonikam, six hundred and sixty-seven; 13 the children of Bigvai, two thousand and sixty-six; 14 the children of Adin, four hundred and fifty-four; 15 the children of Ater, of Hezekiah, ninety-eight; 16 the children of Kilan and Azetas, sixty-seven; 17 the children of Azaru, four hundred and thirty-two; 18 the children of Annias, one hundred and one; 19 the children of Arom and Bethasmoth, forty-two; 20 the children of Kiriathiarius, Kaphirah, and Beroth, seven hundred and forty-three; 21 the children of Ammidioi, Kiriathsarbi, and Hasadias, four hundred and twenty-two; 22 the children of Bethlomon, one hundred and twenty-three; 23 the men of Netophah, fifty-six; 24 the men of Bethasmoth, forty-two; 25 the men of Kiriath-arim, Kephirah, and Beeroth, seven hundred and forty-three; 26 the men of Ramah and Geba, six hundred and twenty-one; 27 the men of Michmas, one hundred and twenty-two; 28 the men of Bethel and Ai, two hundred and twenty-three; 29 the children of Nebo, fifty-two; 30 the children of Magbish, one hundred and fifty-six; 31 the children of the other Elam, one thousand two hundred and fifty-four; 32 the children of Harim, three hundred and twenty; 33 the children of Lod, Hadid, and Ono, seven hundred and twenty-five; 34 the children of Jericho, three hundred and forty-five; 35 the children of Senaah, three thousand six hundred and thirty. 36 The priests: the children of Jedaiah, of the house of Jeshua, nine hundred and seventy-three; 37 the children of Immer, one thousand and fifty-two; 38 the children of Pashhur, one thousand two hundred and forty-seven; 39 the children of Harim, one thousand and seventeen. 40 The Levites: the children of Jeshua and Kadmiel, of the children of Hodaviah, seventy-four. 41 The singers: the children of Asaph, one hundred and twenty-eight. 42 The children of the doorkeepers: the children of Shallum, the children of Ater, the children of Talmon, the children of Akkub, the children of Hatita, the children of Shobai, in all one hundred and thirty-nine. 43 The temple servants: the children of Ziha, the children of Hasupha, the children of Tabbaoth, 44 the children of Keros, the children of Siaha, the children of Padon, 45 the children of Lebanah, the children of Hagabah, the children of Akkub, 46 the children of Hagab, the children of Shalmai, the children of Hanan, 47 the children of Giddel, the children of Gahar, the children of Reaiah, 48 the children of Rezin, the children of Nekoda, the children of Gazzam, 49 the children of Uzza, the children of Paseah, the children of Besai, 50 the children of Asnah, the children of Meunim, the children of Nephussim, 51 the children of Bakbuk, the children of Hakupha, the children of Harhur, 52 the children of Bazlith, the children of Mehida, the children of Harsha, 53 the children of Barkos, the children of Sisera, the children of Thamah, 54 the children of Neziah, the children of Hatipha. 55 The children of the servants of Solomon: the children of Sotai, the children of Sophereth, the children of Peruda, 56 the children of Jaala, the children of Darkon, the children of Giddel, 57 the children of Shephatiah, the children of Hattil, the children of Pochereth-hazzebaim, the children of Ami. 58 All the temple servants and the children of the servants of Solomon were three hundred and seventy-two. 59 And these were those who came up from Tel-melah, Tel-harsha, Cherub, Addan, and Immer, but they could not declare their father's house and their family, whether they were of Israel: 60 the children of Delaiah, the children of Tobiah, the children of Nekoda, six hundred and fifty-two. 61 And of the priests: the children of Hobaiah, the children of Hakkoz, the children of Barzillai, who took a wife from the daughters of Barzillai the Gileadite and was called by their name. 62 These sought their registration among those who were reckoned by genealogy, but they were not found, and they were excluded from the priesthood. 63 And the governor told them that they should not eat of the most holy things until there arose a priest with Urim and Thummim. 64 The whole congregation together was forty-two thousand three hundred and sixty, 65 besides their male servants and female servants, of whom there were seven thousand three hundred and thirty-seven; and among them were two hundred singing men and singing women. 66 Their horses were seven hundred and thirty-six, their mules two hundred and forty-five, 67 their camels four hundred and thirty-five, their asses six thousand seven hundred and twenty. 68 And some of the heads of families, when they came to the house of the Lord which is in Jerusalem, offered willingly for the house of God, to restore it in its place. 69 According to their power they gave to the treasury for the work sixty-one thousand darics of gold, five thousand minas of silver, and one hundred priestly garments. 70 So the priests and the Levites and some of the people and the singers and the doorkeepers and the temple servants lived in their cities, and all Israel in their villages.

3 1 When the seventh month came, and the children of Israel were in their villages, the people gathered together as one man in the open square before the first gate that looks toward the east. 2 Then stood up Jeshua the son of Jozadak and his brothers the priests, and Zerubbabel the son of Shealtiel and his brothers, and they built the altar of the God of Israel, 3 to offer burnt offerings upon it, as it is written in the law of Moses the man of God. 4 And they set the altar upon its foundations, for fear was upon them because of the peoples of the lands, and they offered sacrifices upon it with a clear conscience, and burnt offerings to the Lord morning and evening. 5 And they kept the feast of tabernacles, as it is written, and offered the daily burnt offerings according to the number required by custom, 6 and the offerings for the continual service, and the offerings for the new moon and for all the consecrated feasts of the Lord. 7 And they gave money to the masons and the carpenters, and food and drink and oil to the Sidonians and Tyrians, to bring cedar trees from Mount Lebanon to the sea of Joppa, according to the grant which they had from Cyrus king of the Persians concerning them. 8 And in the second year after they had come to the house of God at Jerusalem, in the second month, Zerubbabel the son of Shealtiel and Jeshua the son of Jozadak and their brothers the priests and the Levites and all who had come to Jerusalem from the captivity began, and appointed the Levites from twenty years old and upward to oversee the work of the house of the Lord. 9 So Jeshua and his sons and brothers, Kadmiel and his sons, the sons of Judah, and the sons of Henadad and their sons and their brothers the Levites, all with one accord undertook the work, overseeing those who were working in the house of God. So the builders built the house of the Lord. 10 And they set the priests in their vestments with trumpets, and the Levites the children of Asaph with cymbals, 11 to praise the Lord, according to the directions of David king of Israel. And they sang with responsive voices, praising and confessing the Lord, "For he is good, for his mercy and truth toward Israel endure forever." And all the people shouted with a great shout when they praised the Lord, because the foundation of the house of the Lord was laid. 12 But many of the priests and Levites and heads of families, old men who had seen the former house, when the foundations of this house were laid before their eyes, wept with a loud voice, 13 while many shouted aloud with joy, so that the people could not discern the sound of the shout of joy from the sound of the weeping of the people, for the people shouted with a great shout, and the sound was heard far away.

4 1 When the enemies of Judah and Benjamin heard that the children of the captivity were building a temple to the Lord God of Israel, 2 they came to Zerubbabel and to the heads of families, and said to them, "Let us build with you, for we seek your God as you do, and we have been sacrificing to him since the days of Esarhaddon king of the Assyrians, who brought us here." 3 But Zerubbabel and Jeshua and the rest of the heads of families of Israel said to them, "It is not for you and us to build a house to our God, but we alone will build to the Lord God of Israel, as King Cyrus the king of the Persians has commanded us." 4 But the peoples of the land hindered the hands of the people of Judah, and troubled them in building, 5 and bribed counselors against them to frustrate their purpose, all the days of Cyrus king of the Persians, even until the reign of Darius king of the Persians.

6 And in the reign of Ahasuerus, in the beginning of his reign, they wrote an accusation against the inhabitants of Judah and Jerusalem. 7 And in the days of Artaxerxes king of the Persians, Bishlam, Mithredath, Tabeel, and the rest of their associates wrote to Artaxerxes king of the Persians; the letter was written in Syrian characters and translated.

8 Rehum the commander and Shimshai the scribe wrote a letter against Jerusalem to Artaxerxes the king as follows: 9 Then Rehum the commander and Shimshai the scribe and their associates, the Dinaites, the Apharsathchites, the Tarpelites, the Apharsites, the Archevites, the Babylonians, the Susanchites, the Dehavites, the Elamites, 10 and the rest of the nations whom the great and glorious Asnapper carried away and settled in the city of Samaria and the rest of the country beyond the River, and so forth. 11 This is a copy of the letter which they sent to him: To Artaxerxes the king, your servants, the men of the country beyond the River, and so forth. 12 Let it be known to the king that the Jews who came up from you to us have gone to Jerusalem, and are building the rebellious and wicked city, and are finishing its marketplaces and repairing its walls. 13 Now let it be known to the king that if this city is built and its walls are finished, they will not pay tribute, custom, or toll, and the royal revenue will suffer. 14 Now because we receive support from the royal palace, it is not right for us to see the king dishonored in this way, and therefore we have sent and informed the king, 15 that search may be made in the book of the records of your fathers. You will find in the book of the records and learn that this city is a rebellious city, troublesome to kings and provinces, and that sedition has been stirred up within it from days of old. For this reason this city was destroyed. 16 We inform the king that if this city is built and its walls are finished, you will not have a portion beyond the River.

17 Then the king sent a word to Rehum the commander and Shimshai the scribe and their associates who dwell in Samaria and the rest beyond the River, and so forth. 18 The letter which you sent to us has been clearly read before me. 19 And I commanded, and search has been made, and it has been found that this city has risen against kings from days of old, and that rebellion and sedition have been fostered in it. 20 There have been mighty kings over Jerusalem, who have ruled over all the country beyond the River, and so forth, and tribute, custom, and toll were paid to them. 21 Now therefore give the command to make these men cease, that this city may not be built, until a command shall be given by me. 22 Take heed not to be slack in this matter. Why should damage grow to the hurt of the kings?

23 Then when the copy of the letter of King Artaxerxes was read before Rehum and Shimshai the scribe and their associates, they went up in haste to Jerusalem to the Jews, and by force and power made them cease. 24 Then the work of the house of God which is in Jerusalem was stopped, and it was discontinued until the second year of the reign of Darius king of the Persians.

5 1 Then Haggai and Zechariah the son of Iddo, prophets, prophesied to the Jews who were in Judea and Jerusalem in the name of the Lord God of Israel, who was over them. 2 Then Zerubbabel the son of Shealtiel and Jeshua the son of Jozadak arose and began to build the house of the Lord which is in Jerusalem, and with them were the prophets of the Lord, helping them. 3 At the same time Tattenai, governor of Syria and Phoenicia, and Shethar-bozenai and their associates came to them and spoke thus to them: Who has commanded you to build this house and to finish this roof and the rest? 4 Then we told them the names of the men who were building this building. 5 But the eye of their God was upon the elders of the Jews, and they did not stop them until a report should go to Darius and a letter be returned concerning it.

6 This is a copy of the letter which Tattenai, governor of Syria and Phoenicia, and Shethar-bozenai and his associates the Apharsachites, who were in Syria and Phoenicia, sent to King Darius. 7 They sent him a report, and in it was written as follows: To Darius the king, all peace. 8 Let it be known to the king that we went into the province of Judea, to the house of the great God, which is being built of hewn stones, and timber is laid in the walls, and this work goes on with diligence and prospers in their hands. 9 Then we asked those elders and spoke thus to them: Who has commanded you to build this house and to finish this roof and the rest? 10 We also asked them their names, to inform you, that we might write the names of the men who were at their head. 11 And this is the word they returned to us, saying: We are the servants of the God of heaven and earth, and we are building the house that was built many years ago, which a great king of Israel built and finished. 12 But after our fathers provoked the God of heaven to wrath, he gave them into the hand of Nebuchadnezzar king of Babylon, the Chaldean, who destroyed this house and carried the people away to Babylon. 13 But in the first year of Cyrus king of Babylon, King Cyrus issued a decree that this house of God should be built. 14 Moreover the gold and silver vessels of the house of God, which Nebuchadnezzar took out of the temple in Jerusalem and carried into the temple of Babylon, King Cyrus took out of the temple of Babylon, and they were delivered to one whose name was Sheshbazzar, whom he had made governor, 15 and he said to him, Take these vessels, go and carry them to the temple in Jerusalem, and let the house of God be built on its site. 16 Then this Sheshbazzar came and laid the foundations of the house of God which is in Jerusalem, and from that time until now it has been building, and it is not yet finished. 17 Now therefore, if it seems good to the king, let search be made among the royal records of the king here in Babylon, whether it is so that a decree was issued by King Cyrus for the building of this house of God in Jerusalem, and let the king send us his decision in this matter.

6 1 Then Darius the king issued a decree, and search was made in Babylon in the archives where the treasures were stored. 2 And a roll was found in Ecbatana, the castle that is in the province of Media, and in it a record was written as follows: 3 In the first year of Cyrus the king, King Cyrus issued a decree concerning the house of God in Jerusalem, that the house be built, the place where sacrifices are offered, and that its foundations be laid firmly, its height sixty cubits and its breadth sixty cubits, 4 with three layers of hewn stones and a layer of timber. Let the cost be paid from the royal treasury. 5 Moreover let the gold and silver vessels of the house of God, which Nebuchadnezzar took out of the temple in Jerusalem and brought to Babylon, be restored and brought to the temple in Jerusalem, each to its place, and you shall put them in the house of God.

6 Now therefore, Tattenai, governor of Syria and Phoenicia, and Shethar-bozenai and your associates the Apharsachites, who are in Syria and Phoenicia, keep far away from there. 7 Let the work on this house of God alone. Let the governor of the Jews and their elders build this house of God on its site. 8 And I issue a decree what you shall do for these elders of the Jews, for the building of this house of God: the cost is to be paid to these men from the royal revenues, from the tribute of Syria and Phoenicia, promptly, so that the work may not be hindered. 9 And what they need, young bulls, rams, or lambs for burnt offerings to the God of heaven, wheat, salt, wine, or oil, according to the requirement of the priests who are in Jerusalem, let that be given them day by day without fail, 10 that they may offer sacrifices of sweet savor to the God of heaven and pray for the life of the king and his children. 11 I have also issued a decree that if any man alters this edict, a beam shall be pulled out of his house, and he shall be impaled on it, and his house shall be confiscated. 12 May the God who has caused his name to dwell there overthrow any king or people who shall put forth a hand to alter this, to destroy this house of God which is in Jerusalem. I Darius have issued a decree; let it be carried out with diligence.

13 Then Tattenai, governor of Syria and Phoenicia, and Shethar-bozenai and their associates carried out with diligence what King Darius had sent. 14 And the elders of the Jews built and prospered through the prophesying of Haggai the prophet and Zechariah the son of Iddo. They built and finished their building by command of the God of Israel and by the command of Cyrus and Darius and Artaxerxes kings of the Persians; 15 and this house was finished on the third day of the month Adar, in the sixth year of the reign of King Darius.

16 Then the children of Israel, the priests and the Levites and the rest of the children of the captivity, celebrated the dedication of this house of God with joy. 17 They offered at the dedication of this house of God one hundred bulls, two hundred rams, four hundred lambs, 18 and twelve he-goats for a sin offering for all Israel, according to the number of the tribes of Israel. 19 And they set the priests in their divisions and the Levites in their courses, for the service of God which is in Jerusalem, according to the book of Moses. 20 The children of the captivity kept the passover on the fourteenth day of the first month, 21 for the priests and the Levites had purified themselves together; all of them were pure. They killed the passover for all the children of the captivity, for their brothers the priests, and for themselves. 22 And the children of Israel who had come out of the captivity ate it, and all who had separated themselves from the pollutions of the peoples of the land and sought the Lord. 23 And they kept the feast of unleavened bread seven days, rejoicing before the Lord, 24 because he had turned the heart of the king of Assyria toward them, to strengthen their hands for the work of the house of God, the God of Israel.

7 1 After these things, in the reign of Artaxerxes king of the Persians, Ezra the son of Seraiah, son of Azariah, son of Hilkiah, 2 son of Shallum, son of Zadok, son of Ahitub, 3 son of Amariah, son of Azariah, son of Meraioth, 4 son of Zerahiah, son of Uzzi, son of Bukki, 5 son of Abishua, son of Phinehas, son of Eleazar, son of Aaron the chief priest— 6 this Ezra went up from Babylon. He was a scribe skilled in the law of Moses which was given by the God of Israel. The king granted him all his request, according to the hand of the Lord his God upon him. 7 And there went up also some of the children of Israel, and some of the priests and Levites, and the singers and doorkeepers and temple servants, to Jerusalem, in the seventh year of Artaxerxes the king. 8 And they came to Jerusalem in the fifth month, which was in the seventh year of the king. 9 For on the first day of the first month he began to go up from Babylon, and on the first day of the fifth month he came to Jerusalem, according to the good hand of his God upon him. 10 For Ezra had set his heart to seek the law of the Lord, and to do it, and to teach in Israel statutes and ordinances.

11 This is a copy of the letter which King Artaxerxes gave to Ezra the priest, the scribe, a scribe of the words of the commandments of the Lord and of his ordinances for Israel: 12 Artaxerxes, king of kings, to Ezra the priest, scribe of the law of the Lord God of heaven, and so forth. 13 I issue a decree that all those of the people of Israel and of their priests and Levites in my realm, who desire of their own free will to go to Jerusalem, may go with you. 14 Inasmuch as you are sent by the king and his seven counselors, to inquire concerning Judea and Jerusalem according to the law of your God which is in your hand, 15 and to carry silver and gold, which the king and his counselors have freely vowed to the God of Israel, whose dwelling is in Jerusalem, 16 and all the silver and gold that you may find in all the province of Babylon, with the freewill offering of the people and of the priests, offering willingly for the house of their God which is in Jerusalem, 17 you shall therefore with all diligence buy with this money bulls, rams, lambs, and their grain offerings and their drink offerings, and offer them upon the altar of the house of your God which is in Jerusalem. 18 And whatever seems good to you and your brothers to do with the rest of the silver and the gold, do according to the will of your God. 19 And the vessels that are given you for the service of the house of your God, deliver before the God of Jerusalem. 20 And whatever more may be required for the house of your God, which it may fall to you to provide, pay it out of the royal treasury. 21 And I, even I Artaxerxes the king, issue a decree to all the treasurers of Syria and Phoenicia, that whatever Ezra the priest, the scribe of the law of the Lord God of heaven, requires of you, let it be done with diligence, 22 up to a hundred talents of silver, and likewise a hundred cors of wheat, a hundred baths of wine, a hundred baths of oil, and salt without prescribing how much. 23 Whatever is commanded by the God of heaven, let it be diligently done for the house of the God of heaven, lest there be wrath against the realm of the king and his sons. 24 We also inform you that it shall not be lawful to impose tribute, custom, or toll on any of the priests or Levites or singers or doorkeepers or temple servants or workmen of this house of God. 25 And you, Ezra, according to the wisdom of your God which is in your hand, appoint magistrates and judges who may judge all the people in Syria and Phoenicia, all such as know the laws of your God; and those who do not know them you shall instruct. 26 And whoever will not obey the law of your God and the law of the king, let judgment be executed upon him with diligence, whether for death or for banishment or for a penalty of money or for imprisonment.

27 Then I Ezra said, Blessed be the Lord God of our fathers, who has put such things as these in the heart of the king, to glorify his house which is in Jerusalem, 28 and has extended mercy to me before the king and his counselors and all his powerful princes. I was encouraged as the hand of the Lord my God was upon me, and I gathered leaders from Israel to go up with me.

8 1 These are the heads of their families and their enrollment of those who went up with me from Babylon in the reign of King Artaxerxes: 2 of the children of Phinehas, Gershom; of the children of Ithamar, Gamael; of the children of David, Hattush 3 son of Shecaniah; of the children of Parosh, Zechariah, and with him were enrolled one hundred and fifty men; 4 of the children of Pahath-moab, Eliehoenai son of Zerahiah, and with him two hundred men; 5 of the children of Shecaniah, the son of Jahaziel, and with him three hundred men; 6 of the children of Adin, Ebed son of Jonathan, and with him two hundred and fifty men; 7 of the children of Elam, Jeshaiah son of Athaliah, and with him seventy men; 8 of the children of Shephatiah, Zebadiah son of Michael, and with him eighty men; 9 of the children of Joab, Obadiah son of Jehiel, and with him two hundred and eighteen men; 10 of the children of Shelomith, the son of Josiphiah, and with him one hundred and sixty men; 11 of the children of Bebai, Zechariah son of Bebai, and with him twenty-eight men; 12 of the children of Azgad, Johanan son of Hakkatan, and with him one hundred and ten men; 13 of the children of Adonikam, the last, whose names are these, Eliphelet, Jeiel, and Shemaiah, and with them sixty men; 14 of the children of Bigvai, Uthai and Zabbud, and with them seventy men.

15 I gathered them by the river that flows to Ahava, and we encamped there three days. When I observed the people and the priests, I found none of the children of Levi there. 16 Then I sent for Eliezer, Ariel, Shemaiah, Elnathan, Jarib, Elnathan, Nathan, Zechariah, and Meshullam, leaders, and for Joiarib and Elnathan, teachers, 17 and I gave them a command for Iddo the chief at the place Casiphia, telling them what they should say to Iddo and his brothers the temple servants at the place Casiphia, that they should bring us ministers for the house of our God. 18 And by the good hand of our God upon us they brought us a man of understanding, of the children of Mahli, son of Levi, son of Israel, namely Sherebiah, with his sons and brothers, eighteen; 19 and Hashabiah, and with him Jeshaiah of the children of Merari, his brothers and their sons, twenty; 20 and of the temple servants, whom David and the princes had appointed for the service of the Levites, two hundred and twenty temple servants, all of them designated by name.

21 Then I proclaimed a fast there at the river Ahava, that we might afflict ourselves before our God, to seek of him a straight way for us, for our children, and for all our property. 22 For I was ashamed to ask the king for troops and horsemen to help us against the enemy on the way, because we had spoken to the king, saying, The hand of our God is upon all who seek him for good, but his power and his wrath are against all who forsake him. 23 So we fasted and sought our God concerning this matter, and he granted our request.

24 Then I separated twelve of the leaders of the priests, Sherebiah, Hashabiah, and ten of their brothers with them, 25 and I weighed out to them the silver and the gold and the vessels, the offering for the house of our God which the king and his counselors and his princes and all Israel there present had offered. 26 I weighed into their hand six hundred and fifty talents of silver, silver vessels worth one hundred talents, one hundred talents of gold, 27 twenty golden bowls worth one thousand darics, and two vessels of fine bronze, precious as gold. 28 And I said to them, You are holy to the Lord, and the vessels are holy, and the silver and the gold are a freewill offering to the Lord God of our fathers. 29 Watch and keep them until you weigh them before the leaders of the priests and the Levites and the princes of the families of Israel at Jerusalem, in the chambers of the house of the Lord. 30 So the priests and the Levites received the silver and the gold and the vessels by weight, to bring them to Jerusalem to the house of our God.

31 Then we departed from the river Ahava on the twelfth day of the first month, to go to Jerusalem. And the hand of our God was upon us, and he delivered us from the hand of every enemy, and we were protected on the way. 32 So we came to Jerusalem and remained there three days. 33 On the fourth day the silver and the gold and the vessels were weighed in the house of our God by the hand of Meremoth son of Uriah the priest, and with him was Eleazar son of Phinehas, and with them were the Levites Jozabad son of Jeshua and Noadiah son of Binnui, 34 by number and by weight of every one. And all the weight was written down at that time. 35 The children of the captivity who had come out of the captivity offered sacrifices to the God of Israel, twelve bulls for all Israel, ninety-six rams, seventy-two lambs, twelve he-goats for a sin offering, all as a burnt offering to the Lord. 36 And they delivered the king's commissions to the king's satraps and to the governors of Syria and Phoenicia, and they helped the people and the house of God.

9 1 Now when these things were done, the princes approached me and said, The people of Israel, the priests and the Levites, have not separated themselves from the peoples of the lands with their abominations, from the Canaanites, Hittites, Perizzites, Jebusites, Ammonites, Moabites, Egyptians, and Edomites. 2 For they have taken some of their daughters for themselves and for their sons, and the holy race has mixed itself with the peoples of the lands. And the hand of the princes and rulers has been foremost in this lawlessness. 3 When I heard this thing, I tore my garment and my robe, and plucked some of the hair from my head and from my beard, and sat down grieving. 4 Then all who trembled at the words of the God of Israel because of the lawlessness of those who had been transported gathered to me, while I sat grieving until the evening sacrifice. 5 Then I arose from my affliction, with my garment and my robe torn, and fell on my knees and spread out my hands to the Lord, 6 and said, O Lord, I am ashamed and blush to lift my face to you, my God, because our sins have increased above our heads, and our lawlessness has grown up to heaven. 7 From the days of our fathers we have been in great lawlessness to this day, and because of our sins we, our kings, and our priests have been delivered into the hand of the kings of the lands, to the sword, to captivity, to plunder, and to shame of face, as it is this day. 8 And now for a little while mercy has been shown by the Lord our God, to leave us a remnant and to give us a stake in the place of his sanctuary, that our God may enlighten our eyes and grant us a little reviving in our bondage. 9 For we are slaves, but in our bondage our God has not forsaken us, but has extended mercy to us before the kings of Persia, to grant us reviving, to exalt the house of our God, to restore its desolations, and to give us a wall in Judea and Jerusalem. 10 And now, what shall we say, O our God, after this? For we have forsaken your commandments, 11 which you commanded by the hand of your servants the prophets, saying, The land which you are entering to possess is a land polluted by the pollutions of the peoples of the lands, and they have filled it with their uncleanness. 12 Now therefore do not give your daughters to their sons, and do not take their daughters for your sons, and do not seek their peace or prosperity forever, that you may be strong and eat the good things of the land and leave it for an inheritance to your children forever. 13 And after all that has come upon us for our evil deeds and our great sins, although you, our God, have punished us less than our sins deserve and have given us such a root, we have returned to transgress your law by intermarrying with the pollutions of the peoples of the lands. 14 Will you not be angry with us to destroy us, so that there should be no remnant or survivor? 15 O Lord God of Israel, you are righteous, for we have been left a remnant, as it is this day. Behold, we are before you in our lawlessness, for because of this no one can stand before you.

10 1 While Ezra prayed and made confession, weeping and prostrating himself before the house of God, there gathered to him from Israel a very great assembly of men and women and children, for the people wept bitterly. 2 Then Shechaniah the son of Jehiel, one of the children of Elam, answered and said to Ezra, We have broken faith with our God and have married foreign women from the peoples of the land, but even now there is hope for Israel in spite of this. 3 Now therefore let us make a covenant with our God to put away all the women and their children, according to the counsel of my lord and of those who tremble at the commandment of our God, and let it be done according to the law. 4 Arise, for this matter is your responsibility, but we will be with you; be strong and act. 5 Then Ezra arose and made the leaders of the priests and the Levites and all Israel swear that they would do according to this word, and they swore. 6 Then Ezra withdrew from before the house of God and went to the chamber of Jehohanan the son of Eliashib, and he lodged there. He ate no bread and drank no water, for he was mourning over the lawlessness of those who had been transported. 7 And a proclamation was made throughout Judea and Jerusalem to all the children of the captivity, that they should gather at Jerusalem, 8 and that whoever did not come within two or three days, according to the decree of the leaders and elders, all his property should be confiscated, and he himself should be excluded from the congregation of those who had been transported. 9 Then all the men of Judah and Benjamin gathered at Jerusalem within the three days. It was the ninth month, on the twentieth day of the month, and all the people sat in the square before the house of God, trembling because of this matter and because of the heavy rain. 10 Then Ezra the priest arose and said to them, You have broken faith and have married foreign women, to increase the sins of Israel. 11 Now therefore make confession to the Lord God of our fathers and do his will, and separate yourselves from the peoples of the land and from the foreign women. 12 Then all the congregation answered and said with a loud voice, It is true, and so we must do. 13 But the people are many, and it is the season of heavy rain, and we are not able to stand outside, and this is not the work of one or two days, for we are many who have committed this lawlessness in this matter. 14 Let our leaders stand for the whole congregation, and let all those in our cities who have married foreign women come at appointed times, and with them the elders of every city and its judges, until the fierce wrath of our God is turned away from us in this matter. 15 Only Jonathan the son of Asahel and Jahzeiah the son of Tikvah opposed this, and Meshullam and Shabbethai the Levite supported them. 16 Then the children of the captivity did so. And Ezra the priest selected men who were heads of families, according to their families, all of them designated by name, and they sat down on the first day of the tenth month to examine the matter. 17 And they finished with all the men who had married foreign women by the first day of the first month.

18 And among the children of the priests who had married foreign women there were found: of the children of Jeshua the son of Jozadak and his brothers, Maaseiah, Eliezer, Jarib, and Gedaliah. 19 And they pledged themselves to put away their wives, and as guilty men to offer a ram of the flock for their guilt. 20 And of the children of Immer, Hanani and Zebadiah. 21 And of the children of Harim, Maaseiah, Elijah, Shemaiah, Jehiel, and Uzziah. 22 And of the children of Pashhur, Elioenai, Maaseiah, Ishmael, Nethanel, Jozabad, and Elasah. 23 And of the Levites, Jozabad, Shimei, Kelaiah (that is, Kelita), Pethahiah, Judah, and Eliezer. 24 And of the singers, Eliashib. And of the doorkeepers, Shallum, Telem, and Uri. 25 And of Israel: of the children of Parosh, Ramiah, Izziah, Malchijah, Mijamin, Eleazar, Malchijah, and Benaiah. 26 And of the children of Elam, Mattaniah, Zechariah, Jehiel, Abdi, Jeremoth, and Elijah. 27 And of the children of Zattu, Elioenai, Eliashib, Mattaniah, Jeremoth, Zabad, and Aziza. 28 And of the children of Bebai, Jehohanan, Hananiah, Zabbai, and Athlai. 29 And of the children of Bani, Meshullam, Malluch, Adaiah, Jashub, Sheal, and Jeremoth. 30 And of the children of Pahath-moab, Adna, Chelal, Benaiah, Maaseiah, Mattaniah, Bezaleel, Binnui, and Manasseh. 31 And of the children of Harim, Eliezer, Ishijah, Malchijah, Shemaiah, Shimeon, 32 Benjamin, Malluch, Shemariah. 33 Of the children of Hashum, Mattenai, Mattattah, Zabad, Eliphelet, Jeremai, Manasseh, and Shimei. 34 Of the children of Bani, Maadai, Amram, Uel, 35 Benaiah, Bedeiah, Chelluh, 36 Vaniah, Meremoth, Eliashib, 37 Mattaniah, Mattenai, Jaasu, 38 Bani, Binnui, Shimei, 39 Shelemiah, Nathan, Adaiah, 40 Macnadebai, Shashai, Sharai, 41 Azarel, Shelemiah, Shemariah, 42 Shallum, Amariah, Joseph. 43 Of the children of Nebo, Jeiel, Mattithiah, Zabad, Zebina, Jaddai, Joel, and Benaiah. 44 All these had married foreign women, and some of these women had children.

Figure 1.1 Illustration of the problem of handwritten digit recognition. How do we get a computer to recognize that these are the digits 504192? That is easy for you to answer, but it is hard for a computer. Neural networks approach the problem in a different way. The idea is to take a large number of handwritten digits, known as training examples, and then develop a system which can learn from those training examples. In other words, the neural network uses the examples to automatically infer rules for recognizing handwritten digits. Furthermore, by increasing the number of training examples, the network can learn more about handwriting, and so improve its accuracy. So while I've shown just 100 training digits above, perhaps we could build a better handwriting recognizer by using thousands or even millions or billions of training examples.

In this chapter we'll write a computer program implementing a neural network that learns to recognize handwritten digits. The program is just 74 lines long, and uses no special neural network libraries. But this short program can recognize digits with an accuracy over 96 percent, without human intervention. Furthermore, in later chapters we'll develop ideas which can improve accuracy to over 99 percent. In fact, the best commercial neural networks are now so good that they are used by banks to process cheques, and by post offices to recognize addresses.

We're focusing on handwriting recognition because it's an excellent prototype problem for learning about neural networks in general. As a prototype it hits a sweet spot: it's challenging - it's no small feat to recognize handwritten digits - but it's not so difficult as to require an extremely complicated solution, or tremendous computational power. Furthermore, it's a good way to develop more advanced techniques, such as deep learning. And so throughout the book we'll return repeatedly to the problem of handwriting recognition. Later in the book, we'll discuss how these ideas may be applied to other problems in computer vision, and also in speech, natural language processing, and other domains.

Of course, if the point of the chapter was only to write a computer program to recognize handwritten digits, then the chapter would be much shorter! But along the way we'll develop many key ideas about neural networks, including two important types of artificial neuron (the perceptron and the sigmoid neuron), and the standard learning algorithm for neural networks, known as stochastic gradient descent. Throughout, I focus on explaining why things are done the way they are, and on building your neural networks intuition. That requires a lengthier discussion than if I just presented the basic mechanics of what's going on, but it's worth it for the deeper understanding you'll attain. Amongst the payoffs, by the end of the chapter we'll be in position to understand what deep learning is, and why it matters.

Perceptrons

What is a neural network? To get started, I'll explain a type of artificial neuron called a perceptron. Perceptrons were developed in the 1950s and 1960s by the scientist Frank Rosenblatt, inspired by earlier work by Warren McCulloch and Walter Pitts. Today, it is more common to use other models of artificial neurons - in this book, and in much modern work on neural networks, the main neuron model used is one called the sigmoid neuron. We'll get to sigmoid neurons shortly. But to understand why sigmoid neurons are defined the way they are, it's worth taking the time to first understand perceptrons.


Mt. Ararat
Lake Van
Lake Urmia
Caspian Sea
Persian Gulf
Arabian Sea
Red Sea
Mediterranean Sea
Black Sea
Sea of Azov
Aral Sea
Lake Balkhash
Lake Baikal
Lake Issyk-Kul
Lake Sevan
Lake Tuz
Lake Van
Lake Urmia
Caspian Sea
Persian Gulf
Arabian Sea
Red Sea
Mediterranean Sea
Black Sea
Sea of Azov
Aral Sea
Lake Balkhash
Lake Baikal
Lake Issyk-Kul
Lake Sevan
Lake Tuz



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Figure 1.1 Sorting as an example of algorithms. The input sequence for each instance of the problem consists of the numbers in brackets; the correct output order is given in brackets. The arrows indicate the transformation from input to output.

For example, given the sequence $\langle 31, 41, 59, 26, 41, 58 \rangle$, a sorting algorithm returns as output the sequence $\langle 26, 31, 41, 41, 58, 59 \rangle$. Such an input sequence is called an *instance* of the sorting problem. In general, an instance of a problem consists of the input (satisfying whatever constraints are imposed in the problem statement) needed to compute a solution to the problem.

Sorting is a fundamental operation in computer science (many programs use it as an intermediate step), and as a result a large number of good sorting algorithms have been developed. Which algorithm is best for a given application depends on—among other factors—the number of items to be sorted, the extent to which the items are already somewhat sorted, possible restrictions on the item values, the architecture of the computer, and the kind of storage devices to be used: main memory, disks, or tapes.

An algorithm is said to be *correct* if, for every input instance, it halts with the correct output. We say that a correct algorithm *solves* the given computational problem. An incorrect algorithm might not halt at all on some input instances, or it might halt with an answer other than the desired one. Contrary to what one might expect, incorrect algorithms can sometimes be useful, if their error rate can be controlled. We shall see an example of this in Chapter 31 when we study algorithms for finding large prime numbers. Ordinarily, however, we shall be concerned only with correct algorithms.

An algorithm can be specified in English, as a computer program, or even as a hardware design. The only requirement is that the specification must provide a precise description of the computational procedure to be followed.

What kinds of problems are solved by algorithms?

Sorting is by no means the only computational problem for which algorithms have been developed. (You probably suspected as much when you saw the size of this book.) Practical applications of algorithms are ubiquitous and include the following examples.

- The Human Genome Project has as its goals the identification of all the 100,000 genes in human DNA, the determination of the sequences of the 3 billion chemical base pairs that make up human DNA, the storage of this information in databases, and the development of tools for data analysis. Each of these steps requires sophisticated algorithms. While the solutions to various problems involved in this project are beyond the scope of this book, many methods have been developed for this biological computing problem that use many of the ideas presented here, thereby enabling scientists to accomplish tasks while using resources efficiently. The savings in time (human and machine) and money that such efficiency provides are enormous.
- The Internet enables people all around the world to quickly access and retrieve large amounts of information. With the aid of clever algorithms, sites on the Internet are able to manage and manipulate this large volume of data. Examples of problems that make essential use of algorithms include finding good routes on which the data will travel (techniques for solving such problems appear in Chapter 24), and using a search engine to quickly find pages on which particular information resides (related techniques are in Chapters 11 and 32).
- Electronic commerce enables goods and services to be negotiated and exchanged electronically, and it depends on the privacy of information such as credit card numbers, passwords, and bank statements. The core technologies used in electronic commerce include public-key cryptography and digital signatures (Chapter 31), which are based on numerical algorithms and number theory.
- Manufacturing and other commercial enterprises often need to allocate scarce resources in the most beneficial way. An oil company may wish to know where to place its wells in order to maximize its expected profit. A political candidate may want to determine where to spend money buying campaign advertising in order to maximize the chances of winning an election. An airline may wish to assign crews to flights in the least expensive way possible, making sure that each flight is covered and that government regulations regarding crew scheduling are met. An Internet service provider may wish to determine where to place additional resources in order to serve its customers more effectively. All of these are examples of problems that can be solved using linear programming, which we shall study in Chapter 29.

While some of the details of these examples are beyond the scope of this book, we do give basic techniques that apply to these problems and problem areas. We also show how to solve many concrete problems, including the following.

- We are given a road map on which the distance between each pair of adjacent intersections is marked, and our goal is to determine the shortest possible route from one intersection to another. The number of possible routes can be huge, even if we disallow routes that cross over themselves. How do we choose which of all possible routes is the shortest? Here, we model the road map (which is itself a model of actual roads) as a graph (which we will meet in Part VI and Appendix B), and we wish to find the shortest path from one vertex in the graph to another. Chapter 24 shows how to solve this problem efficiently.
- We are given a mechanical design in terms of a library of parts, where each part may include instances of other parts, and we need to list the parts in order so that we will see the definition of each part before we see any part that uses it. If the design consists of n parts, then we can easily verify that an ordering can be found in time n^2. Although this is the best we can do in the worst case, we shall see how to do much better in more typical cases. Chapter 22 gives the details.
- We are given a set of numerical constraints, and we wish to find a solution that satisfies them (or determine that no solution exists). For example, if we are given constraints that require x to be at least 2 and y to be at least 6, and that require $x + y$ to be at most 7, then one solution is $x = 2$ and $y = 5$. Another solution is $x = 2$ and $y = 6$. Yet another solution is $x = 3$ and $y = 4$. Although this example has only two variables, problems with thousands of variables can arise in practice. Linear programming, which we shall study in Chapter 29, gives us a way to solve this problem.

Data structures

This book also contains several data structures. A *data structure* is a way to store and organize data in order to facilitate access and modifications. No single data structure works well for all purposes, so it is important to know the strengths and limitations of several of them.

Technique

Although you can use this book as a "cookbook" for algorithms, you may at some point find that no algorithm in this book does what you need done, or that you don't understand the algorithm that you need. Then you may have to design your own algorithm, or modify an existing algorithm from this book to fit your needs. The remainder of this book will help you to understand algorithms and techniques for designing them.

Hard problems

Most of this book is about efficient algorithms. Our usual measure of efficiency is speed, i.e., how long an algorithm takes to produce its result. There are some problems, however, for which no efficient solution is known. Chapter 34 studies an interesting subset of these problems, which are known as NP-complete.

Why are NP-complete problems interesting? First, although no efficient algorithm has ever been discovered for an NP-complete problem, nobody has ever proven that an efficient algorithm for one cannot exist. In other words, it is unknown whether efficient algorithms exist for NP-complete problems. Second, the set of NP-complete problems has the remarkable property that if an efficient algorithm exists for any one of them, then efficient algorithms exist for all of them. This relationship among the NP-complete problems makes the lack of efficient solutions all the more tantalizing. Third, several NP-complete problems are similar, but not identical, to problems for which we do know of efficient algorithms. A small change to the problem statement can cause a big change to the efficiency of the best known algorithm.

It is valuable to know about NP-complete problems because some of them arise surprisingly often in real applications. If you are called upon to produce an efficient algorithm for an NP-complete problem, you are likely to spend a lot of time in fruitless searching. If you can instead demonstrate that the problem is NP-complete, you can spend your time developing an efficient algorithm that gives a good, but not the best possible, solution.

As a concrete example, consider a trucking company with a central depot. A truck must travel from the depot to each of several addresses to deliver goods. Each truck can travel only a limited distance, because it must carry enough fuel for the trip. We want to find a route that visits all the addresses, provided that the truck has enough fuel for the route. This problem is closely related to the traveling-salesman problem, which is NP-complete, and we do not know of an efficient algorithm for it. Suppose that you have a fuel-efficient truck that can travel only a distance d between refueling, and that the distance between each pair of addresses is marked on the map. Then it is easy to find a solution that minimizes the number of times that the truck refuels: just modify the breadth-first-search algorithm in Section 22.2 so that it starts from the depot and stops when it reaches the first address that is more than d units away. Although this algorithm does not necessarily minimize the total distance traveled, it is likely to be good in practice. Proving that the algorithm has this property for all inputs would be difficult, however, because if it did, it would also solve the traveling-salesman problem efficiently.

Exercises

1.1-1 Give a real-world example that requires sorting or a real-world example that requires computing a convex hull.

1.1-2 Other than speed, what other measures of efficiency might one use in a real-world setting?

1.1-3 Select a data structure that you have seen previously, and discuss its strengths and limitations.

1.1-4 How are the shortest-path and traveling-salesman problems given above similar? How are they different?

1.1-5 Suggest a real-world problem in which only the best solution will do. Then come up with one in which approximately the best solution is good enough.

1.2 Algorithms as a technology

Suppose computers were infinitely fast and computer memory was free. Would you have any reason to study algorithms? The answer is yes, if for no other reason than that you would still like to demonstrate that your solution method terminates and does so with the correct answer.

If computers were infinitely fast, any correct method for solving a problem would do. You would probably want your implementation to be within the bounds of good software engineering practice (i.e., well designed and documented), but you would usually choose the method that is easiest to implement.

Of course, computers may be fast, but they are not infinitely fast. And memory may be cheap, but it is not free. Computing time is therefore a bounded resource, and so is space in memory. These resources should be used wisely, and algorithms that are efficient in terms of time or space will help you do so.

Efficiency

Different algorithms devised to solve the same problem often differ dramatically in their efficiency. These differences can be much more significant than differences due to hardware and software.

As an example, in Chapter 2, we will see two algorithms for sorting. The first, known as insertion sort, takes time roughly equal to $c_1 n^2$ to sort n items, where c_1 is a constant that does not depend on n. That is, it takes time roughly proportional to n^2. The second, merge sort, takes time roughly equal to $c_2 n \lg n$, where $\lg n$ stands for $\log_2 n$ and c_2 is another constant that also does not depend on n. Insertion sort usually has a smaller constant factor than merge sort, so that $c_1 < c_2$. We shall see that the constant factors can have far less impact on the running time than the dependence on the input size n. Let's write insertion sort's running time as $c_1 n \cdot n$ and merge sort's running time as $c_2 n \lg n$. Then we see that where insertion sort has a factor of n in its running time, merge sort has a factor of $\lg n$, which is much smaller. (For example, when $n = 1000$, $\lg n$ is approximately 10, and when n is 1 million, $\lg n$ is approximately only 20.) Although insertion sort usually runs faster than merge sort for small input sizes, once the input size n becomes large enough, merge sort's advantage of $\lg n$ vs. n will more than compensate for the difference in constant factors. No matter how much smaller c_1 is than c_2, there will always be a crossover point beyond which merge sort is faster.

For a concrete example, let us pit a faster computer (computer A) running insertion sort against a slower computer (computer B) running merge sort. They must each sort an array of 1 million numbers. Suppose that computer A executes 1 billion instructions per second and computer B executes only 10 million instructions per second, so that computer A is 100 times faster than computer B in raw computing power. To make the difference even more dramatic, suppose that the world's craftiest programmer codes insertion sort in machine language for computer A, and the resulting code requires $2n^2$ instructions to sort n numbers. Suppose further that just an average programmer writes merge sort for computer B, using a high-level language with an inefficient compiler, with the resulting code taking $50 n \lg n$ instructions. To sort 1 million numbers, computer A takes

$$\frac{2 \cdot (10^6)^2 \text{ instructions}}{10^9 \text{ instructions/second}} = 2000 \text{ seconds},$$

while computer B takes

$$\frac{50 \cdot 10^6 \lg 10^6 \text{ instructions}}{10^7 \text{ instructions/second}} \approx 100 \text{ seconds}.$$

By using an algorithm whose running time grows more slowly, even with a poor compiler, computer B runs 20 times faster than computer A! The advantage of merge sort is even more pronounced when we sort 10 million numbers: where insertion sort takes approximately 2.3 days, merge sort takes under 20 minutes. In general, as the problem size increases, so does the relative advantage of merge sort.

Algorithms and other technologies

The example above shows that algorithms, like computer hardware, are a technology. Total system performance depends on choosing efficient algorithms as much as on choosing fast hardware. Just as rapid advances are being made in other computer technologies, they are being made in algorithms as well.

You might wonder whether algorithms are truly that important on contemporary computers in light of other advanced technologies, such as

- hardware with high clock rates, pipelining, and superscalar architectures,
- easy-to-use, intuitive graphical user interfaces (GUIs),
- object-oriented systems, and
- integrated Web technologies.

The answer is yes. Although some applications do not explicitly require algorithmic content at the application level (e.g., some simple Web-based applications), most also require a certain amount of algorithmic content themselves. For example, suppose that a Web service determines how to get from one location to another (as several do). Its implementation would likely rely on hardware, a GUI, wide-area network connectivity, and perhaps object orientation. It would also require algorithms for such operations as rendering maps, interpolating addresses, and finding routes. Algorithms based on the techniques in this book would probably be needed to provide routes, since this service would have to find routes among many locations.

Even an application that does not require algorithmic content at the application level relies heavily upon algorithms. Does the application rely on hardware to get its work done? Then algorithms are used in the hardware design. Does the application run in a GUI environment? Then algorithms are used in the GUI implementation. Does the application operate over a network? Then algorithms are used in the network routing hardware. Does the application process text? Then algorithms are used for text processing. Clearly, algorithms are a technology that underlies most of computer science.

Exercises

1.2-1 Give an example of an application that requires algorithmic content at the application level, and discuss the function of the algorithms involved.

1.2-2 Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \lg n$ steps. For which values of n does insertion sort beat merge sort? How might one rearrange the pseudocode for merge sort to obtain better performance?

1.2-3 What is the smallest value of n such that an algorithm whose running time is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine?

Problems

1-1 Comparison of running times

For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes $f(n)$ microseconds.

	1 second	1 minute	1 hour	1 day	1 month	1 year	1 century
$\lg n$							
$\sqrt{n}$							
n							
$n \lg n$							
n^2							
n^3							
2^n							
$n!$							

1-2 Insertion sort on small arrays in merge sort

Although merge sort runs in $\Theta(n \lg n)$ worst-case time and insertion sort runs in $\Theta(n^2)$ worst-case time, the constant factors in insertion sort can make it faster in practice for small problem sizes on many machines. Thus it makes sense to coarsen the leaves of the recursion by using insertion sort within merge sort when subproblems become sufficiently small. Consider a modification to merge sort in which n/k sublists of length k are sorted using insertion sort and then merged using the standard merging mechanism, where k is a value to be determined.

a. Show that insertion sort can sort the n/k sublists, each of length k, in $\Theta(nk)$ worst-case time.

b. Show how to merge the sublists in $\Theta(n \lg(n/k))$ worst-case time.

c. Given that the modified algorithm runs in $\Theta(nk + n \lg(n/k))$ worst-case time, what is the largest value of k as a function of n for which the modified algorithm has the same running time as standard merge sort, in terms of Θ-notation?

d. How should we choose k in practice?

1-3 Inversions

Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an *inversion* of A.

a. List the five inversions of the array $\langle 2, 3, 8, 6, 1 \rangle$.

b. What array with elements from the set $\{1, 2, \ldots, n\}$ has the most inversions? How many does it have?

c. What is the relationship between the running time of insertion sort and the number of inversions in the input array? Justify your answer.

d. Give an algorithm that determines the number of inversions in any permutation on n elements in $\Theta(n \lg n)$ worst-case time. (*Hint:* Modify merge sort.)

Chapter notes

There are many excellent texts on the general topic of algorithms, including those by Aho, Hopcroft, and Ullman [5, 6], Baase and Van Gelder [28], Brassard and Bratley [46], Goodrich and Tamassia [151], Horowitz, Sahni, and Rajasekaran [176], Kingston [200], Knuth [204, 205, 207], Kozen [211], Manber [232], Mehlhorn [246, 247, 248], Purdom and Brown [280], Reingold, Nievergelt, and Deo [287], Sedgewick [303], Sedgewick and Flajolet [304], Skiena [317], Smith [320], Tarjan [337], Udi Manber [339], Wilf [358], and Williamson [359]. Some of the more practical aspects of algorithm design are discussed by Bentley [37, 38] and Gonnet [147].

Chapter 3 Sample Size Determination 45
3.1 Introduction 45
3.2 Sample Size Determination for Estimating a Mean 46
3.3 Sample Size Determination for Estimating a Proportion 48
3.4 Sample Size Determination for Comparing Two Means 50
3.5 Sample Size Determination for Comparing Two Proportions 52
3.6 Power Analysis 54
3.7 Sample Size Determination for Finite Populations 56
3.8 Sample Size Determination for Cluster Sampling 58
3.9 Sample Size Determination for Stratified Sampling 60
3.10 Sample Size Determination for Systematic Sampling 62
3.11 Sample Size Determination for Multistage Sampling 64
3.12 Sample Size Determination for Ratio Estimation 66
3.13 Sample Size Determination for Regression Estimation 68
3.14 Sample Size Determination for Difference Estimation 70
3.15 Sample Size Determination for Proportion Estimation 72
3.16 Sample Size Determination for Mean Estimation 74
3.17 Sample Size Determination for Total Estimation 76
3.18 Sample Size Determination for Variance Estimation 78
3.19 Sample Size Determination for Standard Deviation Estimation 80
3.20 Sample Size Determination for Coefficient of Variation Estimation 82
3.21 Sample Size Determination for Correlation Estimation 84
3.22 Sample Size Determination for Regression Coefficient Estimation 86
3.23 Sample Size Determination for Intraclass Correlation Estimation 88
3.24 Sample Size Determination for Design Effect Estimation 90
3.25 Sample Size Determination for Cost Optimization 92
3.26 Sample Size Determination for Time Optimization 94
3.27 Sample Size Determination for Resource Optimization 96
3.28 Sample Size Determination for Multiple Objectives 98
3.29 Sample Size Determination for Multiple Parameters 100
3.30 Sample Size Determination for Multiple Populations 102
3.31 Sample Size Determination for Multiple Strata 104
3.32 Sample Size Determination for Multiple Clusters 106
3.33 Sample Size Determination for Multiple Stages 108
3.34 Sample Size Determination for Multiple Phases 110
3.35 Sample Size Determination for Multiple Waves 112
3.36 Sample Size Determination for Multiple Panels 114
3.37 Sample Size Determination for Multiple Rotations 116
3.38 Sample Size Determination for Multiple Subsamples 118
3.39 Sample Size Determination for Multiple Replicates 120
3.40 Sample Size Determination for Multiple Imputations 122
3.41 Sample Size Determination for Multiple Weights 124
3.42 Sample Size Determination for Multiple Adjustments 126
3.43 Sample Size Determination for Multiple Corrections 128
3.44 Sample Size Determination for Multiple Comparisons 130
3.45 Sample Size Determination for Multiple Tests 132
3.46 Sample Size Determination for Multiple Models 134
3.47 Sample Size Determination for Multiple Variables 136
3.48 Sample Size Determination for Multiple Outcomes 138
3.49 Sample Size Determination for Multiple Predictors 140
3.50 Sample Size Determination for Multiple Interactions 142
3.51 Sample Size Determination for Multiple Nonlinearities 144
3.52 Sample Size Determination for Multiple Missing Data Mechanisms 146
3.53 Sample Size Determination for Multiple Measurement Errors 148
3.54 Sample Size Determination for Multiple Sources of Variation 150
3.55 Sample Size Determination for Multiple Sources of Bias 152
3.56 Sample Size Determination for Multiple Sources of Uncertainty 154
3.57 Sample Size Determination for Multiple Sources of Error 156
3.58 Sample Size Determination for Multiple Sources of Nonresponse 158
3.59 Sample Size Determination for Multiple Sources of Attrition 160
3.60 Sample Size Determination for Multiple Sources of Contamination 162
3.61 Sample Size Determination for Multiple Sources of Misclassification 164
3.62 Sample Size Determination for Multiple Sources of Measurement Error 166
3.63 Sample Size Determination for Multiple Sources of Sampling Error 168
3.64 Sample Size Determination for Multiple Sources of Nonsampling Error 170
3.65 Sample Size Determination for Multiple Sources of Total Error 172
3.66 Sample Size Determination for Multiple Sources of Mean Square Error 174
3.67 Sample Size Determination for Multiple Sources of Root Mean Square Error 176
3.68 Sample Size Determination for Multiple Sources of Mean Absolute Error 178
3.69 Sample Size Determination for Multiple Sources of Mean Absolute Percentage Error 180
3.70 Sample Size Determination for Multiple Sources of Mean Absolute Scaled Error 182
3.71 Sample Size Determination for Multiple Sources of Mean Absolute Deviation 184
3.72 Sample Size Determination for Multiple Sources of Median Absolute Deviation 186
3.73 Sample Size Determination for Multiple Sources of Interquartile Range 188
3.74 Sample Size Determination for Multiple Sources of Range 190
3.75 Sample Size Determination for Multiple Sources of Variance 192
3.76 Sample Size Determination for Multiple Sources of Standard Deviation 194
3.77 Sample Size Determination for Multiple Sources of Coefficient of Variation 196
3.78 Sample Size Determination for Multiple Sources of Correlation 198
3.79 Sample Size Determination for Multiple Sources of Regression Coefficient 200
3.80 Sample Size Determination for Multiple Sources of Intraclass Correlation 202
3.81 Sample Size Determination for Multiple Sources of Design Effect 204
3.82 Sample Size Determination for Multiple Sources of Cost 206
3.83 Sample Size Determination for Multiple Sources of Time 208
3.84 Sample Size Determination for Multiple Sources of Resource 210
3.85 Sample Size Determination for Multiple Sources of Objective 212
3.86 Sample Size Determination for Multiple Sources of Parameter 214
3.87 Sample Size Determination for Multiple Sources of Population 216
3.88 Sample Size Determination for Multiple Sources of Stratum 218
3.89 Sample Size Determination for Multiple Sources of Cluster 220
3.90 Sample Size Determination for Multiple Sources of Stage 222
3.91 Sample Size Determination for Multiple Sources of Phase 224
3.92 Sample Size Determination for Multiple Sources of Wave 226
3.93 Sample Size Determination for Multiple Sources of Panel 228
3.94 Sample Size Determination for Multiple Sources of Rotation 230
3.95 Sample Size Determination for Multiple Sources of Subsample 232
3.96 Sample Size Determination for Multiple Sources of Replicate 234
3.97 Sample Size Determination for Multiple Sources of Imputation 236
3.98 Sample Size Determination for Multiple Sources of Weight 238
3.99 Sample Size Determination for Multiple Sources of Adjustment 240
3.100 Sample Size Determination for Multiple Sources of Correction 242





Franklin was now in his twenty-first year, and had attained a considerable degree of skill in the art of printing. He had also written several pieces in prose and verse, which had been published in the paper of his brother. Of these, the most remarkable was a burlesque upon the style of the Mathers, which had been inserted in a pamphlet published by Cotton Mather, and which had given great offence to the people of Boston. The author of this burlesque was discovered, and the displeasure of the people was turned against him. He was summoned before the House of Assembly, and committed to prison for a month. This circumstance, which at the time was a source of great mortification to him, afterwards proved the means of his rising to eminence in his native country.

Franklin's brother, James, was the first person in Boston who published a newspaper. It was called the New England Courant, and was the fourth newspaper published in America. The first was published in Boston in 1690, the second in Philadelphia in 1719, and the third in Boston in 1721. The New England Courant was published in 1721. Franklin was employed in printing this paper, and also in writing articles for it. His brother was the editor, but Franklin was the principal writer. Many of his pieces were published in the paper, and were much admired by the readers. They were written in imitation of the style of the Spectator, a celebrated work published in London, which Franklin had read with great attention and delight.

Franklin's brother was a man of a violent temper, and often treated him with severity. This was the principal reason why Franklin left his service before the term of his apprenticeship had expired. He was determined to go to New York, and seek employment in the printing-house of Mr. Bradford, the first printer who had established himself in that city. But a vessel was about to sail for that place from Newport, and he went on board of her, and worked his passage. He was a stranger in Newport, and knew no one there. He was obliged to sell his clothes to pay for his passage. He had a few books with him, which he sold to a bookseller in Newport. He then went to New York, and found employment in the printing-house of Mr. Bradford.

Figure 1.1 Illustration of the problem of handwritten digit recognition. How do we get a computer to recognize that these are the digits 504192? That is easy for you to answer, but it is hard for a computer. Neural networks approach the problem in a different way. The idea is to take a large number of handwritten digits, known as training examples, and then develop a system which can learn from those training examples. In other words, the neural network uses the examples to automatically infer rules for recognizing handwritten digits. Furthermore, by increasing the number of training examples, the network can learn more about handwriting, and so improve its accuracy. So while I've shown just 100 training digits above, perhaps we could build a better handwriting recognizer by using thousands or even millions or billions of training examples.

In this chapter we'll write a computer program implementing a neural network that learns to recognize handwritten digits. The program is just 74 lines long, and uses no special neural network libraries. But this short program can recognize digits with an accuracy over 96 percent, without human intervention. Furthermore, in later chapters we'll develop ideas which can improve accuracy to over 99 percent. In fact, the best commercial neural networks are now so good that they are used by banks to process cheques, and by post offices to recognize addresses.

We're focusing on handwriting recognition because it's an excellent prototype problem for learning about neural networks in general. As a prototype it hits a sweet spot: it's challenging - it's no small feat to recognize handwritten digits - but it's not so difficult as to require an extremely complicated solution, or tremendous computational power. Furthermore, it's a great way to develop more advanced techniques, such as deep learning. And so throughout the book we'll return repeatedly to the problem of handwriting recognition. Later in the book, we'll discuss how these ideas may be applied to other problems in computer vision, and also in speech, natural language processing, and other domains.

Of course, if the point of the chapter was only to write a computer program to recognize handwritten digits, then the chapter would be much shorter! But along the way we'll develop many key ideas about neural networks, including two important types of artificial neuron (the perceptron and the sigmoid neuron), and the standard learning algorithm for neural networks, known as stochastic gradient descent. Throughout, I focus on explaining why things are done the way they are, and on building your neural networks intuition. That requires a lengthier discussion than if I just presented the basic mechanics of what's going on, but it's worth it for the deeper understanding you'll attain. Amongst the payoffs, by the end of the chapter we'll be in position to understand what deep learning is, and why it matters.

Perceptrons

What is a neural network? To get started, I'll explain a type of artificial neuron called a perceptron. Perceptrons were developed in the 1950s and 1960s by the scientist Frank Rosenblatt, inspired by earlier work by Warren McCulloch and Walter Pitts. Today, it is more common to use other models of artificial neurons - in this book, and in much modern work on neural networks, the main neuron model used is one called the sigmoid neuron. We'll get to sigmoid neurons shortly. But to understand why sigmoid neurons are defined the way they are, it's worth taking the time to first understand perceptrons.


Ethernet Header
IP Header
TCP Header
Application Data
Ethernet Trailer

Ferrous sulfate is a mild astringent and is used in the treatment of minor skin irritations. It is also used as a source of iron in the treatment of iron deficiency anemia.

Ch1
Ch2
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Ch39
Ch40
Ch41

Lemma 2.1. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^n$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then

$$\chi(\mathcal{A}, t) = \chi(\mathcal{A} \setminus \mathcal{B}, t) - \chi(\mathcal{A}^{\mathcal{B}}, t).$$

Proof. Let $\mathcal{B} = \{H_1, \ldots, H_k\}$. Then

$$\chi(\mathcal{A}, t) = \chi(\mathcal{A} \setminus \{H_1\}, t) - \chi(\mathcal{A}^{H_1}, t)$$

$$= \chi(\mathcal{A} \setminus \{H_1, H_2\}, t) - \chi(\mathcal{A}^{H_1}, t) - \chi(\mathcal{A}^{H_2}, t) + \chi(\mathcal{A}^{H_1 \cap H_2}, t)$$

$$= \cdots$$

$$= \chi(\mathcal{A} \setminus \mathcal{B}, t) - \sum_{\emptyset \neq \mathcal{C} \subseteq \mathcal{B}} (-1)^{|\mathcal{C}|} \chi(\mathcal{A}^{\mathcal{C}}, t).$$

Now apply Theorem 2.1 to the last term. $\square$

Theorem 2.2. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^n$. Then

$$\chi(\mathcal{A}, t) = \sum_{\mathcal{B} \subseteq \mathcal{A}} (-1)^{|\mathcal{B}|} t^{n - \operatorname{rank}(\mathcal{B})}.$$

Proof. Induction on $|\mathcal{A}|$. If $\mathcal{A} = \emptyset$, then $\chi(\mathcal{A}, t) = t^n$, as desired. Now suppose $\mathcal{A} \neq \emptyset$, and let $H \in \mathcal{A}$. By Lemma 2.1,

$$\chi(\mathcal{A}, t) = \chi(\mathcal{A} \setminus \{H\}, t) - \chi(\mathcal{A}^H, t).$$

By induction,

$$\chi(\mathcal{A} \setminus \{H\}, t) = \sum_{\mathcal{B} \subseteq \mathcal{A} \setminus \{H\}} (-1)^{|\mathcal{B}|} t^{n - \operatorname{rank}(\mathcal{B})}$$

and

$$\chi(\mathcal{A}^H, t) = \sum_{\mathcal{B} \subseteq \mathcal{A}^H} (-1)^{|\mathcal{B}|} t^{\dim H - \operatorname{rank}(\mathcal{B})}.$$

Now $\operatorname{rank}(\mathcal{B}) = \operatorname{rank}(\mathcal{B} \cup \{H\}) - 1$ for $\mathcal{B} \subseteq \mathcal{A}^H$, and $\dim H = n - 1$. Hence

$$\chi(\mathcal{A}^H, t) = \sum_{\mathcal{B} \subseteq \mathcal{A}^H} (-1)^{|\mathcal{B}|} t^{n - \operatorname{rank}(\mathcal{B} \cup \{H\})}.$$

Thus

$$\chi(\mathcal{A}, t) = \sum_{\mathcal{B} \subseteq \mathcal{A} \setminus \{H\}} (-1)^{|\mathcal{B}|} t^{n - \operatorname{rank}(\mathcal{B})} + \sum_{\mathcal{B} \subseteq \mathcal{A}^H} (-1)^{|\mathcal{B} \cup \{H\}|} t^{n - \operatorname{rank}(\mathcal{B} \cup \{H\})}$$

$$= \sum_{\mathcal{B} \subseteq \mathcal{A}} (-1)^{|\mathcal{B}|} t^{n - \operatorname{rank}(\mathcal{B})}. \quad \square$$

Corollary 2.1. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^n$. Then

$$\chi(\mathcal{A}, t) = \sum_{i=0}^{n} (-1)^i w_i(\mathcal{A}) t^{n-i},$$

where $w_i(\mathcal{A})$ is the number of i-dimensional subspaces in $L(\mathcal{A})$.

Proof. Let $\mathcal{B} \subseteq \mathcal{A}$ and $\operatorname{rank}(\mathcal{B}) = i$. Then $\bigcap \mathcal{B}$ is an i-dimensional subspace in $L(\mathcal{A})$. Conversely, if X is an i-dimensional subspace in $L(\mathcal{A})$, then $\mathcal{A}^X$ is a Boolean algebra of rank i with X as its unique minimal element. Hence there are $w_i(\mathcal{A})$ subsets $\mathcal{B} \subseteq \mathcal{A}$ with $\operatorname{rank}(\mathcal{B}) = i$ and $\bigcap \mathcal{B} = X$. Now apply Theorem 2.2. $\square$

Corollary 2.2. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^n$. Then

$$r(\mathcal{A}) = \sum_{i=0}^{n} w_i(\mathcal{A}).$$

Proof. Let $\mathcal{B} \subseteq \mathcal{A}$ and $\operatorname{rank}(\mathcal{B}) = i$. Then $\bigcap \mathcal{B}$ is an i-dimensional subspace in $L(\mathcal{A})$. Conversely, if X is an i-dimensional subspace in $L(\mathcal{A})$, then $\mathcal{A}^X$ is a Boolean algebra of rank i with X as its unique minimal element. Hence there are $w_i(\mathcal{A})$ subsets $\mathcal{B} \subseteq \mathcal{A}$ with $\operatorname{rank}(\mathcal{B}) = i$ and $\bigcap \mathcal{B} = X$. Now apply Theorem 2.2. $\square$

Society is a complex organism, and its complexity is the result of the complexity of the individuals who compose it. Each individual is a complex organism, and his complexity is the result of the complexity of the cells which compose him. Each cell is a complex organism, and its complexity is the result of the complexity of the molecules which compose it. Each molecule is a complex organism, and its complexity is the result of the complexity of the atoms which compose it. Each atom is a complex organism, and its complexity is the result of the complexity of the electrons which compose it. Each electron is a complex organism, and its complexity is the result of the complexity of the ether which composes it. Each ether is a complex organism, and its complexity is the result of the complexity of the space which composes it. Each space is a complex organism, and its complexity is the result of the complexity of the time which composes it. Each time is a complex organism, and its complexity is the result of the complexity of the eternity which composes it. Each eternity is a complex organism, and its complexity is the result of the complexity of the God which composes it. Each God is a complex organism, and its complexity is the result of the complexity of the universe which composes it. Each universe is a complex organism, and its complexity is the result of the complexity of the multiverse which composes it. Each multiverse is a complex organism, and its complexity is the result of the complexity of the omniverse which composes it. Each omniverse is a complex organism, and its complexity is the result of the complexity of the pluriverse which composes it. Each pluriverse is a complex organism, and its complexity is the result of the complexity of the unicosm which composes it. Each unicosm is a complex organism, and its complexity is the result of the complexity of the multicosm which composes it. Each multicosm is a complex organism, and its complexity is the result of the complexity of the omnicosm which composes it. Each omnicosm is a complex organism, and its complexity is the result of the complexity of the pluricosm which composes it. Each pluricosm

Mr. Wiley's method of determining the amount of sugar in sorghum juice is as follows:

"Fifty cubic centimeters of the juice are measured into a flask and diluted with water to 100 cubic centimeters. Ten cubic centimeters of Fehling's solution are measured into a porcelain dish and diluted with water to 40 cubic centimeters. The dish is placed upon a wire gauze and heated to boiling. The diluted juice is then added from a burette, a little at a time, until the blue color of the solution disappears. The number of cubic centimeters of juice required to reduce the 10 cubic centimeters of Fehling's solution is noted. Ten cubic centimeters of Fehling's solution are reduced by 0.05 gram of invert sugar. Hence the number of cubic centimeters of juice required to reduce 10 cubic centimeters of Fehling's solution contains 0.05 gram of invert sugar. The amount of sugar in the juice is then calculated."

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In order to determine the exact nature of the Fe-Ni-Co alloy, a sample was analyzed by X-ray diffraction. The diffraction pattern showed the presence of a single phase with a face-centered cubic (FCC) crystal structure. The lattice parameter was calculated to be 3.589 Å, which is consistent with the value reported for the Invar alloy Fe-36Ni. The absence of any additional peaks in the diffraction pattern indicates that the cobalt is fully dissolved in the FCC lattice and does not form a separate phase.

The thermal expansion coefficient of the alloy was measured using a dilatometer. The sample was heated from room temperature to 100°C at a rate of 5°C per minute. The linear expansion coefficient was found to be 1.2×10^{-6} per °C, which is approximately one order of magnitude lower than that of pure iron. This low thermal expansion coefficient is the key property that makes Invar alloys useful for applications where dimensional stability is required.

The magnetic properties of the alloy were also investigated. The Curie temperature, which is the temperature above which the material loses its ferromagnetic properties, was determined to be 230°C. This is slightly lower than the Curie temperature of pure nickel, which is 358°C. The addition of cobalt to the Fe-Ni alloy has the effect of reducing the Curie temperature, which can be advantageous in some applications.

The mechanical properties of the alloy were tested using a tensile testing machine. The yield strength was found to be 280 MPa, and the ultimate tensile strength was 480 MPa. The elongation at fracture was 35%, indicating that the material is ductile. These values are typical for Invar alloys and are sufficient for most engineering applications.

In conclusion, the Fe-Ni-Co alloy prepared in this study exhibits the characteristic low thermal expansion coefficient of Invar alloys, along with a single-phase FCC crystal structure and good mechanical properties. The addition of cobalt has the effect of reducing the Curie temperature, which may be beneficial in certain applications. Further studies are planned to investigate the effect of different cobalt concentrations on the properties of the alloy.

Knights Companions of the Most Noble Order of the Garter, and to the Officers of Arms, and to the Gentleman Usher of the Black Rod, and to the Gentleman Usher of the Green Rod, and to the Gentleman Usher of the Scarlet Rod, and to the Gentleman Usher of the Blue Rod, and to the Gentleman Usher of the Purple Rod, and to the Gentleman Usher of the Red Rod, and to the Gentleman Usher of the Orange Rod, and to the Gentleman Usher of the Yellow Rod, and to the Gentleman Usher of the White Rod, and to the Gentleman Usher of the Brown Rod, and to the Gentleman Usher of the Grey Rod, and to the Gentleman Usher of the Pink Rod, and to the Gentleman Usher of the Violet Rod, and to the Gentleman Usher of the Crimson Rod, and to the Gentleman Usher of the Maroon Rod, and to the Gentleman Usher of the Lavender Rod, and to the Gentleman Usher of the Turquoise Rod, and to the Gentleman Usher of the Magenta Rod, and to the Gentleman Usher of the Cerise Rod, and to the Gentleman Usher of the Mauve Rod, and to the Gentleman Usher of the Fawn Rod, and to the Gentleman Usher of the Buff Rod, and to the Gentleman Usher of the Ecru Rod, and to the Gentleman Usher of the Sepia Rod, and to the Gentleman Usher of the Umber Rod, and to the Gentleman Usher of the Sienna Rod, and to the Gentleman Usher of the Terra Cotta Rod, and to the Gentleman Usher of the Raw Umber Rod, and to the Gentleman Usher of the Burnt Umber Rod, and to the Gentleman Usher of the Raw Sienna Rod, and to the Gentleman Usher of the Burnt Sienna Rod, and to the Gentleman Usher of the Yellow Ochre Rod, and to the Gentleman Usher of the Red Ochre Rod, and to the Gentleman Usher of the Brown Ochre Rod, and to the Gentleman Usher of the Green Ochre Rod, and to the Gentleman Usher of the Blue Ochre Rod, and to the Gentleman Usher of the Purple Ochre Rod, and to the Gentleman Usher of the Orange Ochre Rod, and to the Gentleman Usher of the Violet Ochre Rod, and to the Gentleman Usher of the Crimson Ochre Rod, and to the Gentleman Usher of the Maroon Ochre Rod, and to the Gentleman Usher of the Lavender Ochre Rod, and to the Gentleman Usher of the Turquoise Ochre Rod, and to the Gentleman Usher of the Magenta Ochre Rod, and to the Gentleman Usher of the Cerise Ochre Rod, and to the Gentleman Usher of the Mauve Ochre Rod, and to the Gentleman Usher of the Fawn Ochre Rod, and to the Gentleman Usher of the Buff Ochre Rod, and to the Gentleman Usher of the Ecru Ochre Rod, and to the Gentleman Usher of the Sepia Ochre Rod, and to the Gentleman Usher of the Umber Ochre Rod, and to the Gentleman Usher of the Sienna Ochre Rod, and to the Gentleman Usher of the Terra Cotta Ochre Rod, and to the Gentleman Usher of the Raw Umber Ochre Rod, and to the Gentleman Usher of the Burnt Umber Ochre Rod, and to the Gentleman Usher of the Raw Sienna Ochre Rod, and to the Gentleman Usher of the Burnt Sienna Ochre

Exercise 2. Lie on the floor, face down, with the hands under the forehead. Raise the head and shoulders as high as possible, keeping the feet on the floor. Hold this position for a count of ten. Relax. Repeat.

Exercise 3. Lie on the floor, face down, with the hands under the forehead. Raise the head and shoulders as high as possible, keeping the feet on the floor. Hold this position for a count of ten. Relax. Repeat.

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Exercise 8. Lie on the floor, face down, with the hands under the forehead. Raise the head and shoulders as high as possible, keeping the feet on the floor. Hold this position for a count of ten. Relax. Repeat.

Exercise 9. Lie on the floor, face down, with the hands under the forehead. Raise the head and shoulders as high as possible, keeping the feet on the floor. Hold this position for a count of ten. Relax. Repeat.

Exercise 10. Lie on the floor, face down, with the hands under the forehead. Raise the head and shoulders as high as possible, keeping the feet on the floor. Hold this position for a count of ten. Relax. Repeat.

Hippocrates' theory of humors was a theory that the body contained four humors, or fluids, that needed to be in balance for a person to be healthy. The four humors were blood, phlegm, yellow bile, and black bile. Hippocrates believed that each humor was associated with a particular organ and that an imbalance in the humors could cause illness. For example, he believed that too much blood could cause fever, while too much phlegm could cause coughing and congestion.

Hippocrates also developed a system of diagnosis and treatment based on his theory of humors. He believed that physicians should examine the patient's body and symptoms to determine which humor was out of balance. Treatment would then be aimed at restoring the balance of the humors. For example, if a patient had a fever, Hippocrates might prescribe bloodletting to reduce the amount of blood in the body.

Hippocrates' theory of humors was influential for many centuries and was the basis of medical practice in Europe and the Middle East until the 19th century. However, it is now known that the theory is not accurate. The humors do not exist, and imbalances in the humors do not cause illness. Nevertheless, Hippocrates' theory of humors was an important step in the development of medicine, as it was the first attempt to explain illness in terms of natural causes rather than supernatural ones.

Figure 1.1 Illustration of the problem of polynomial curve fitting, showing the data set and the function $\sin(2\pi x)$.

$$y(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + \ldots + w_M x^M = \sum_{j=0}^{M} w_j x^j \tag{1.1}$$

where M is the order of the polynomial, and x^j denotes x raised to the power of j. The polynomial coefficients $w_0, \ldots, w_M$ are collectively denoted by the vector $\mathbf{w}$. Note that, although the polynomial function $y(x, \mathbf{w})$ is a nonlinear function of x, it is a linear function of the coefficients $\mathbf{w}$. Functions, such as the polynomial, which are linear in the unknown parameters have important properties and are called *linear models*.

The values of the coefficients will be determined by fitting the polynomial to the training data. This can be done by minimizing an *error function* that measures the misfit between the function $y(x, \mathbf{w})$, for any given value of $\mathbf{w}$, and the training set data points. One simple choice of error function, which is widely used, is given by the sum of the squares of the errors between the predictions $y(x_n, \mathbf{w})$ for each data point x_n and the corresponding target value t_n, so that we minimize

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2 \tag{1.2}$$

where the factor of $\frac{1}{2}$ is included for later convenience. We shall discuss the motivation for this choice of error function later in this chapter. For the moment we simply note that it is a nonnegative quantity that would be zero if, and only if, the function $y(x, \mathbf{w})$ were to pass exactly through each training data point. The geometrical interpretation of the sum-of-squares error function is illustrated in Figure 1.2.

Figure 1.2 The error function corresponds to (one half of) the sum of the squares of the displacements (shown by the vertical green bars) of each data point from the function $y(x, \mathbf{w})$.

We can solve the curve fitting problem by choosing the value of $\mathbf{w}$ for which $E(\mathbf{w})$ is as small as possible. Because the error function is a quadratic function of the coefficients $\mathbf{w}$, its derivatives with respect to the coefficients will be linear in the elements of $\mathbf{w}$, and so the minimization of the error function has a unique solution, denoted by $\mathbf{w}^\star$, which can be found in closed form. The resulting polynomial is given by the function $y(x, \mathbf{w}^\star)$.

There remains the problem of choosing the order M of the polynomial, and as we shall see this will turn out to be an example of an important concept called *model comparison* or *model selection*. In Figure 1.4, we show four examples of the results of fitting polynomials having orders $M = 0$, $M = 1$, $M = 3$, and $M = 9$ to the data set shown in Figure 1.2.

We notice that the constant ($M = 0$) and first order ($M = 1$) polynomials give rather poor fits to the data and consequently rather poor representations of the function $\sin(2\pi x)$. The third order ($M = 3$) polynomial seems to give the best fit to the function $\sin(2\pi x)$ of the examples shown in Figure 1.4. When we go to a much higher order polynomial ($M = 9$), we obtain an excellent fit to the training data. In fact, the polynomial passes exactly through each data point and $E(\mathbf{w}^\star) = 0$. However, the fitted curve oscillates wildly and gives a very poor representation of the function $\sin(2\pi x)$. This latter behaviour is known as *over-fitting*.

As we have noted earlier, the goal is to achieve good generalization by making accurate predictions for new data. We can obtain some quantitative insight into the dependence of the generalization performance on M by considering a separate test set comprising 100 data points generated using exactly the same procedure used to generate the training set points but with new choices for the random noise values included in the target values. For each choice of M, we can then evaluate the residual value of $E(\mathbf{w}^\star)$ given by (1.2) for the training data, and we can also evaluate $E(\mathbf{w}^\star)$ for the test data set. It is sometimes more convenient to use the *root-mean-square* (RMS) error defined by

$$E_{\mathrm{RMS}} = \sqrt{2E(\mathbf{w}^\star)/N} \tag{1.3}$$

in which the division by N allows us to compare different sizes of data sets on an equal footing, and the square root ensures that E_{RMS} is measured on the same scale (and in the same units) as the target variable t. Graphs of the training and test set RMS errors are shown, for various values of M, in Figure 1.5. The test set error is a measure of how well we are doing in predicting the values of t for new data observations of x. We note from Figure 1.5 that small values of M give relatively large values of the test set error, and this can be attributed to the fact that the corresponding polynomials are rather inflexible and are incapable of capturing the oscillations in the function $\sin(2\pi x)$. Values of M in the range $3 \leq M \leq 8$ give small values for the test set error, and these also give reasonable representations of the generating function $\sin(2\pi x)$, as can be seen, for the case of $M = 3$, from Figure 1.4.

Figure 1.4 Plots of polynomials having various orders M, shown as red curves, fitted to the data set shown in Figure 1.2.

Figure 1.5 Graph of the root-mean-square error, defined by (1.3), evaluated on the training set and on an independent test set for various values of M.

For $M = 9$, the training set error goes to zero, as we might expect because this polynomial contains 10 degrees of freedom corresponding to the 10 coefficients $w_0, \ldots, w_9$, and so can be tuned exactly to the 10 data points in the training set. However, the test set error has become very large and, as we saw in Figure 1.4, the corresponding function $y(x, \mathbf{w}^\star)$ exhibits wild oscillations.

This may seem paradoxical because a polynomial of given order contains all lower order polynomials as special cases. The $M = 9$ polynomial is therefore capable of generating results at least as good as the $M = 3$ polynomial. Furthermore, we might suppose that the best predictor of new data would be the function $\sin(2\pi x)$ from which the data was generated (and we shall see later that this is indeed the case). We know that a power series expansion of the function $\sin(2\pi x)$ contains terms of all orders, so we might expect that results should improve monotonically as we increase M.

We can gain some insight into the problem by examining the values of the coefficients $\mathbf{w}^\star$ obtained from polynomials of various order, as shown in Table 1.1. We see that, as M increases, the magnitude of the coefficients typically gets larger. In particular for the $M = 9$ polynomial, the coefficients have become finely tuned to the data by developing large positive and negative values so that the corresponding polynomial function matches each of the data points exactly, but between data points (particularly near the ends of the range) the function exhibits the large oscillations observed in Figure 1.4. Intuitively, what is happening is that the more flexible polynomials with larger values of M are becoming increasingly tuned to the random noise on the target values.

Table 1.1 Table of the coefficients $\mathbf{w}^\star$ for polynomials of various order. Observe how the typical magnitudes of the coefficients increase dramatically as the order of the polynomial increases.

	$M = 0$	$M = 1$	$M = 3$	$M = 9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		−1.27	7.99	232.37
$w_2^\star$			−25.43	−5321.83
$w_3^\star$			17.37	48568.31
$w_4^\star$				−231639.30
$w_5^\star$				640042.26
$w_6^\star$				−1061800.52
$w_7^\star$				1042400.18
$w_8^\star$				−557682.99
$w_9^\star$				125201.43

It is also interesting to examine the behaviour of a given model as the number of data points is varied. Figure 1.6 shows the result of fitting the $M = 9$ polynomial to 15 data points (left plot) and to 100 data points (right plot). We see that the larger data set is able to support the more complex model and that the over-fitting is not as severe.

Figure 1.6 Plots of the solutions obtained by minimizing the sum-of-squares error function using the $M = 9$ polynomial for $N = 15$ data points (left plot) and $N = 100$ data points (right plot). We see that increasing the size of the data set reduces the over-fitting problem.

Over-fitting is an unfortunate property of maximum likelihood and does not arise when we marginalize over parameters in a Bayesian setting. We shall see that the Bayesian approach can lead to models with many more parameters than the number of data points, without over-fitting.

Over-fitting is also an example of a phenomenon called *bias* and is related to the problem of *over-confidence* that we shall encounter shortly. Intuitively, over-fitting occurs when we have fitted our data too well and are capturing the noise as well as the signal that is inherent in the data. Given a finite data set, the model parameters are often tuned to the particular details of the training data, rather than to the more general properties from which we want our model to make predictions.

We can obtain a deeper insight into the over-fitting problem by examining how the model parameters behave as we vary the size of the data set. From Figure 1.6 we see that, for a given model complexity, the over-fitting problem becomes less severe as the size of the data set increases. Another way to say this is that the larger the data set, the more complex (in other words more flexible) the model that we can afford to fit to the data. One rough heuristic that is sometimes advocated is that the number of data points should be no less than some multiple (say 5 or 10) of the number of adaptive parameters in the model. However, as we shall see in Chapter 3, the number of parameters is not necessarily the most appropriate measure of model complexity.

Also, there should be no difficulty in fitting models which have a greater number of parameters than the number of data points. More complex models may be used naturally to tackle larger problems. Section 3.4 will show that Bayesian models are unavoidably biased and that the bias will typically be reduced as the size of the data set is increased for a fixed number of parameters. Unfortunately, it will also be seen that in practice the increase in the number of parameters usually far outstrips any benefit deriving from the reduction in bias.

Techniques such as maximum likelihood can lead to over-fitting because they involve finding a single point in parameter space (a value of $\mathbf{w}$). Consideration of the corresponding distribution over $\mathbf{w}$, as provided by the Bayesian approach, can avoid over-fitting and will lead to automatic methods for determining the appropriate level of complexity for the model.

1.1.1 Regularization

We have seen that, in order to obtain good generalization, we may need to limit the complexity of the model in order to avoid over-fitting. This can also be done by adding a penalty term to the error function (1.2) in order to discourage the coefficients from reaching large values. The simplest such penalty term takes the form of a sum of squares of all of the coefficients, leading to a modified error function of the form

$$\widetilde{E}(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2} \|\mathbf{w}\|^2 \tag{1.4}$$

where $\|\mathbf{w}\|^2 \equiv \mathbf{w}^{\mathrm{T}}\mathbf{w} = w_0^2 + w_1^2 + \ldots + w_M^2$, and the coefficient λ governs the relative importance of the regularization term compared with the sum-of-squares error term. Note that often the coefficient w_0 is omitted from the regularizer because its inclusion causes the results to depend on the choice of origin for the target variable (Hastie *et al.*, 2001), or it may be included but with its own regularization coefficient (we shall discuss this topic in more detail in Section 5.5.1). Again, the error function in (1.4) can be minimized exactly in closed form. Techniques such as this are known in the statistics literature as *shrinkage* methods because they reduce the value of the coefficients. The particular case of a quadratic regularizer is called *ridge regression* (Hoerl and Kennard, 1970). In the context of neural networks, this approach is known as *weight decay*.

Figure 1.7 shows the results of fitting the $M = 9$ polynomial to the same data set as used previously but now using the regularized error function given by (1.4). We see that, for a value of $\ln \lambda = -18$, the over-fitting has been suppressed and we now obtain a much closer representation of the underlying function $\sin(2\pi x)$. If, however, we use too large a value for λ then we obtain poor results, as shown in Figure 1.7 for the case of $\ln \lambda = 0$. The corresponding coefficients from the fitted polynomials are given in Table 1.2, showing that regularization has the desired effect of reducing the magnitude of the coefficients.

Figure 1.7 Plots of $M = 9$ polynomials fitted to the data set shown in Figure 1.2 using the regularized error function (1.4) for two values of the regularization parameter λ corresponding to $\ln \lambda = -18$ and $\ln \lambda = 0$. The case of no regularizer, i.e., $\lambda = 0$, corresponding to $\ln \lambda = -\infty$, is shown in Figure 1.4.

Table 1.2 Table of the coefficients $\mathbf{w}^\star$ for $M = 9$ polynomials for various values of the regularization parameter λ. Note that $\ln \lambda = -\infty$ corresponds to a model with no regularization, i.e., to the graph shown in the bottom right plot of Figure 1.4. We see that, as the value of λ increases, the typical magnitude of the coefficients gets smaller.

	$\ln \lambda = -\infty$	$\ln \lambda = -18$	$\ln \lambda = 0$
$w_0^\star$	0.35	0.35	0.13
$w_1^\star$	232.37	4.74	−0.05
$w_2^\star$	−5321.83	−0.77	−0.06
$w_3^\star$	48568.31	−31.97	−0.05
$w_4^\star$	−231639.30	−3.89	−0.03
$w_5^\star$	640042.26	55.28	−0.02
$w_6^\star$	−1061800.52	41.32	−0.01
$w_7^\star$	1042400.18	−45.95	−0.00
$w_8^\star$	−557682.99	−91.53	0.00
$w_9^\star$	125201.43	72.68	0.01

The impact of the regularization term on the generalization error can be seen by plotting the value of the RMS error (1.3) for both training and test sets against $\ln \lambda$, as shown in Figure 1.8. We see that in effect λ now controls the effective complexity of the model and hence determines the degree of over-fitting.

Figure 1.8 Graph of the root-mean-square error (1.3) evaluated on the training set and on the test set for various values of $\ln \lambda$ for the $M = 9$ polynomial.

The issue of model complexity is an important one and will be discussed at length in Section 1.3. Here we simply note that, if we were trying to solve a practical application using this approach of minimizing an error function, we would have to find a way to determine a suitable value for the model complexity. The results above suggest a simple way of achieving this, namely by taking the available data and partitioning it into a training set, used to determine the coefficients $\mathbf{w}$, and a separate validation set, also called a hold-out set, used to optimize the model complexity (either M or λ). In many cases, however, this will prove to be too wasteful of valuable training data, and we have to seek more sophisticated approaches.

So far our discussion of polynomial curve fitting has appealed largely to intuition. We now seek a more principled approach to solving problems in pattern recognition by turning to a discussion of probability theory. As well as providing the foundation for nearly all of the subsequent developments in this book, it will also give us some important insights into the assumptions and approximations that have been made in the previous discussion.

Figure 1.1 Overview of the chapters in this book.

Chapter 2 introduces the basic concepts of machine learning, including the learning process, model evaluation, and model selection. Chapter 3 introduces the basic concepts of deep learning, including the development of neural networks, the structure of neural networks, and the training of neural networks. Chapter 4 introduces the basic concepts of computer vision, including the development of computer vision, the tasks of computer vision, and the applications of computer vision. Chapter 5 introduces the basic concepts of natural language processing, including the development of natural language processing, the tasks of natural language processing, and the applications of natural language processing. Chapter 6 introduces the basic concepts of speech recognition, including the development of speech recognition, the tasks of speech recognition, and the applications of speech recognition. Chapter 7 introduces the basic concepts of reinforcement learning, including the development of reinforcement learning, the tasks of reinforcement learning, and the applications of reinforcement learning. Chapter 8 introduces the basic concepts of generative models, including the development of generative models, the tasks of generative models, and the applications of generative models. Chapter 9 introduces the basic concepts of transfer learning, including the development of transfer learning, the tasks of transfer learning, and the applications of transfer learning. Chapter 10 introduces the basic concepts of multi-task learning, including the development of multi-task learning, the tasks of multi-task learning, and the applications of multi-task learning. Chapter 11 introduces the basic concepts of meta-learning, including the development of meta-learning, the tasks of meta-learning, and the applications of meta-learning. Chapter 12 introduces the basic concepts of explainable AI, including the development of explainable AI, the tasks of explainable AI, and the applications of explainable AI. Chapter 13 introduces the basic concepts of AI ethics, including the development of AI ethics, the tasks of AI ethics, and the applications of AI ethics. Chapter 14 introduces the basic concepts of AI safety, including the development of AI safety, the tasks of AI safety, and the applications of AI safety. Chapter 15 introduces the basic concepts of AI governance, including the development of AI governance, the tasks of AI governance, and the applications of AI governance. Chapter 16 introduces the basic concepts of AI future, including the development of AI future, the tasks of AI future, and the applications of AI future.

1.3 Summary

In this chapter, we have introduced the basic concepts of artificial intelligence, including the definition of artificial intelligence, the development of artificial intelligence, and the applications of artificial intelligence. We have also introduced the structure of this book, including the chapters and their relationships. In the next chapter, we will introduce the basic concepts of machine learning, including the learning process, model evaluation, and model selection.



Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

Fig. 11.

Fig. 12.

Fig. 13.

Fig. 14.

Fig. 15.

Fig. 16.

Fig. 17.

Fig. 18.

Fig. 19.

Fig. 20.

Fig.





Abstract. We consider the problem of finding the optimal control of a stochastic system with a finite number of states. The system is described by a Markov chain with a transition probability matrix that depends on the control. The objective is to minimize the expected total cost over a finite horizon. We show that the optimal control can be found by solving a system of linear equations. We also discuss the case of an infinite horizon and the convergence of the solution.

1991 Mathematics Subject Classification: 93E20, 60J10.

1. Introduction

Let $\{X_n, n = 0, 1, 2, \ldots\}$ be a Markov chain with state space $S = \{1, 2, \ldots, N\}$ and transition probability matrix $P(u) = (p_{ij}(u))$, where $u \in U$ is a control parameter. The cost function is given by

$$C(x, u) = \sum_{i=1}^{N} c_i(u) \mathbb{1}_{\{x=i\}},$$

where $c_i(u)$ is the cost of being in state i when control u is applied.

We define the value function

$$V_n(x) = \min_{u \in U} \left\{ C(x, u) + \sum_{j=1}^{N} p_{xj}(u) V_{n-1}(j) \right\}, \quad n = 1, 2, \ldots$$

with $V_0(x) = 0$ for all $x \in S$.

1.1 Preliminaries

We assume that the set U is finite. For each $x \in S$, let $u^*(x)$ be a control that attains the minimum in the definition of $V_n(x)$. Then the optimal control at time n is $u^*(X_n)$.

1.2 Main result

Theorem 1. The value function $V_n(x)$ satisfies the system of linear equations

$$V_n(x) = C(x, u^*(x)) + \sum_{j=1}^{N} p_{xj}(u^*(x)) V_{n-1}(j), \quad x = 1, 2, \ldots, N.$$

Proof. By definition of $u^*(x)$, we have

$$V_n(x) \leq C(x, u) + \sum_{j=1}^{N} p_{xj}(u) V_{n-1}(j) \quad \text{for all } u \in U.$$

Taking $u = u^*(x)$ gives the result. $\square$

2. Infinite horizon

We now consider the infinite horizon problem with discount factor $\beta \in (0, 1)$. The value function is

$$V(x) = \min_{u \in U} \left\{ C(x, u) + \beta \sum_{j=1}^{N} p_{xj}(u) V(j) \right\}.$$

Theorem 2. The value function $V(x)$ is the unique solution to the system of linear equations

$$V(x) = C(x, u^*(x)) + \beta \sum_{j=1}^{N} p_{xj}(u^*(x)) V(j), \quad x = 1, 2, \ldots, N.$$

Proof. The proof is similar to that of Theorem 1 and is omitted. $\square$

3. Conclusion

We have shown that the optimal control can be found by solving a system of linear equations. This is a well-known result in the theory of Markov decision processes.

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of the Society, and the Council have the power to remove any Fellow from the Society by a majority of two-thirds of the Councillors present at a meeting specially called for the purpose, provided that the Fellow has been previously informed of the charges against him, and has had an opportunity of being heard in his defence.

The Council may also remove any Fellow who has failed to pay his annual contribution for three years, after having given him notice of the arrears due by him, and of the intention of the Council to remove him from the Society, unless he shall pay the same within a year from the date of such notice.

Any Fellow removed from the Society may be restored to his former rank by the Council, on his paying the arrears due by him at the time of his removal, or such part thereof as the Council may think proper to remit.

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Dr. Dana's Classification of the Crustacea.

Dr. Dana's classification of the Crustacea is based upon the following principles:—

1. The Crustacea are divided into two primary groups, the Thoracostraca and the Arthrostraca, according as the thoracic segments are united with the head to form a cephalothorax, or are free.

2. The Thoracostraca are divided into two orders, the Podophthalma and the Edriophthalma, according as the eyes are pedunculated or sessile.

3. The Podophthalma are divided into two suborders, the Decapoda and the Stomapoda, according as the thoracic feet are ten or more than ten.

4. The Decapoda are divided into two tribes, the Macrura and the Brachyura, according as the abdomen is long or short.

5. The Macrura are divided into two sections, the Anomala and the Normalia, according as the abdomen is asymmetrical or symmetrical.

6. The Anomala are divided into two families, the Paguridea and the Thalassinidea, according as the abdomen is soft or hard.

7. The Normalia are divided into two families, the Astacidea and the Scyllaridea, according as the antennæ are pediform or setiform.

8. The Brachyura are divided into two sections, the Oxystomata and the Eurystomata, according as the mouth is narrow or broad.

9. The Oxystomata are divided into two families, the Leptopodidea and the Oxyrhynchidea, according as the feet are slender or stout.

10. The Eurystomata are divided into two families, the Cancridea and the Catometopidea, according as the eyes are lateral or frontal.

11. The Stomapoda are divided into two families, the Squillidea and the Erichthidea, according as the abdomen is long or short.

12. The Edriophthalma are divided into two suborders, the Amphipoda and the Isopoda, according as the abdomen is pedunculated or sessile.

13. The Amphipoda are divided into two tribes, the Gammaridea and the Caprellidea, according as the abdomen is long or short.

14. The Isopoda are divided into two tribes, the Cymothoadea and the Oniscidea, according as the abdomen is symmetrical or asymmetrical.

15. The Cymothoadea are divided into two sections, the Flabellifera and the Natatoria, according as the feet are flabelliform or natatorial.

16. The Flabellifera are divided into two families, the Cymothoidea and the Idoteidea, according as the antennæ are setiform or pediform.

17. The Natatoria are divided into two families, the Anthuridea and the Serolidea, according as the abdomen is short or long.

18. The Oniscidea are divided into two sections, the Epicaridea and the Scutocoidea, according as the abdomen is short or long.

19. The Epicaridea are divided into two families, the Bopyridea and the Entoniscidea, according as the abdomen is symmetrical or asymmetrical.

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Ancient Greek letters in context: a cognitive-historical approach to lexical change. Cambridge: Cambridge University Press. doi: 10.1017/9781139381376

Abstract

Ancient Greek letters in context is

Lemma 2.1. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}| = |\mathcal{A}'|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.2. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{=}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{=}| = |\mathcal{A}'^{=}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{=}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{=}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.3. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.4. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.5. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.6. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\neq}| = |\mathcal{A}'^{\neq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\neq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.7. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.8. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.9. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.10. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\neq}| = |\mathcal{A}'^{\neq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\neq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.11. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.12. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.13. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.14. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\neq}| = |\mathcal{A}'^{\neq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\neq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.15. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.16. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.17. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.18. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\neq}| = |\mathcal{A}'^{\neq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\neq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.19. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.20. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.21. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.22. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\neq}| = |\mathcal{A}'^{\neq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\neq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.23. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.24. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.25. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.26. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\neq}| = |\mathcal{A}'^{\neq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\neq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.27. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.28. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.29. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.30. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\neq}| = |\mathcal{A}'^{\neq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\neq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.31. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.32. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.33. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.34. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\neq}| = |\mathcal{A}'^{\neq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\neq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.35. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{\geq}| = |\mathcal{A}'^{\geq}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{\geq}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{\geq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.36. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{<}| = |\mathcal{A}'^{<}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{<}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{<}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.37. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}'$ be the set of hyperplanes obtained from $\mathcal{A}$ by moving each hyperplane in $\mathcal{B}$ by a small amount in the direction of its normal vector. Then $|\mathcal{A}^{>}| = |\mathcal{A}'^{>}| + |\mathcal{B}^{=}|$ and $|\mathcal{A}^{\leq}| = |\mathcal{A}'^{\leq}|$. Since $\mathcal{A}'$ is a set of hyperplanes in general position, $|\mathcal{A}'^{>}| \leq |\mathcal{A}'^{\leq}|$. Therefore, $|\mathcal{A}^{>}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Lemma 2.38. Let $\mathcal{A}$ be a finite set of affine hyperplanes in $\mathbb{R}^d$ and let $\mathcal{B}$ be a subset of $\mathcal{A}$. Then $|\mathcal{A}^{\neq}| \leq |\mathcal{A}^{\leq}| + |\mathcal{B}^{=}|$.

Proof. Let $\mathcal{A}

Cuando el hombre se encuentra en el mundo, se encuentra en un mundo de cosas, de objetos, de instrumentos, de utensilios, de herramientas, de medios. Y se encuentra en un mundo de otros hombres, de prójimos, de semejantes, de hermanos, de enemigos, de amigos, de adversarios, de competidores, de colaboradores, de cooperadores, de rivales, de contrincantes, de antagonistas, de aliados, de socios, de asociados, de compañeros, de camaradas, de colegas, de condiscípulos, de correligionarios, de conciudadanos, de compatriotas, de paisanos, de vecinos, de parientes, de familiares, de consanguíneos, de padres, de madres, de hijos, de hijas, de hermanos, de hermanas, de tíos, de tías, de primos, de primas, de sobrinos, de sobrinas, de abuelos, de abuelas, de nietos, de nietas, de bisabuelos, de bisabuelas, de tatarabuelos, de tatarabuelas, de suegros, de suegras, de yernos, de nueras, de cuñados, de cuñadas, de padrastros, de madrastras, de hijastros, de hijastras, de padrinos, de madrinas, de ahijados, de ahijadas, de compadres, de comadres, de amigos íntimos, de amigos del alma, de amigos del corazón, de amigos de la infancia, de amigos de la juventud, de amigos de la adolescencia, de amigos de la niñez, de amigos de la escuela, de amigos del colegio, de amigos de la universidad, de amigos del trabajo, de amigos del club, de amigos del barrio, de amigos del pueblo, de amigos de la ciudad, de amigos del país, de amigos del mundo, de amigos de todos los tiempos, de amigos de todas las épocas, de amigos de todas las edades, de amigos de todas las razas, de amigos de todas las religiones, de amigos de todas las ideologías, de amigos de todas las clases sociales, de amigos de todas las profesiones, de amigos de todas las nacionalidades, de amigos de todos los continentes, de amigos de todos los países, de amigos de todas las ciudades, de amigos de todos los pueblos, de amigos de todos los barrios, de amigos de todos los clubes, de amigos de todos los trabajos, de amigos de todas las universidades, de amigos de todos los colegios, de amigos de todas las escuelas, de amigos de todas las niñeces, de amigos de todas las adolescencias, de amigos de todas las juventudes, de amigos de todas las infancias, de amigos de todas las épocas, de amigos de todas las edades, de amigos de todas las razas, de amigos de todas las religiones, de amigos de todas las ideologías, de amigos de todas las clases sociales, de amigos de todas las profesiones, de amigos de todas las nacionalidades, de amigos de todos los continentes, de amigos de todos los países, de amigos de todas las ciudades, de amigos de todos los pueblos, de amigos de todos los barrios, de amigos de todos los clubes, de amigos de todos los trabajos, de amigos de todas las universidades, de amigos de todos los colegios, de amigos de todas las escuelas, de amigos de todas las niñeces, de amigos de todas las adolescencias, de amigos de todas las juventudes, de amigos de todas las infancias.

Figure 1.1 Two computers connected by a network

Figure 1.2 Three computers connected by a network

Figure 1.3 Four computers connected by a network

Figure 1.4 Five computers connected by a network

Figure 1.5 Six computers connected by a network

Figure 1.6 Seven computers connected by a network

Figure 1.7 Eight computers connected by a network

Figure 1.8 Nine computers connected by a network

Figure 1.9 Ten computers connected by a network

Figure 1.10 Eleven computers connected by a network

Figure 1.11 Twelve computers connected by a network

Figure 1.12 Thirteen computers connected by a network

Figure 1.13 Fourteen computers connected by a network

Figure 1.14 Fifteen computers connected by a network

Figure 1.15 Sixteen computers connected by a network

Figure 1.16 Seventeen computers connected by a network

Figure 1.17 Eighteen computers connected by a network

Figure 1.18 Nineteen computers connected by a network

Figure 1.19 Twenty computers connected by a network

Figure 1.20 Twenty-one computers connected by a network

Figure 1.21 Twenty-two computers connected by a network

Figure 1.22 Twenty-three computers connected by a network

Figure 1.23 Twenty-four computers connected by a network

Figure 1.24 Twenty-five computers connected by a network

Figure 1.25 Twenty-six computers connected by a network

Figure 1.26 Twenty-seven computers connected by a network

Figure 1.27 Twenty-eight computers connected by a network

Figure 1.28 Twenty-nine computers connected by a network

Figure 1.29 Thirty computers connected by a network

Figure 1.30 Thirty-one computers connected by a network

Figure 1.31 Thirty-two computers connected by a network

Figure 1.32 Thirty-three computers connected by a network

Figure 1.33 Thirty-four computers connected by a network

Figure 1.34 Thirty-five computers connected by a network

Figure 1.35 Thirty-six computers connected by a network

Figure 1.36 Thirty-seven computers connected by a network

Figure 1.37 Thirty-eight computers connected by a network

Figure 1.38 Thirty-nine computers connected by a network

Figure 1.39 Forty computers connected by a network

Figure 1.40 Forty-one computers connected by a network

Figure 1.41 Forty-two computers connected by a network

Figure 1.42 Forty-three computers connected by a network

Figure 1.43 Forty-four computers connected by a network

Figure 1.44 Forty-five computers connected by a network

Figure 1.45 Forty-six computers connected by a network

Figure 1.46 Forty-seven computers connected by a network

Figure 1.47 Forty-eight computers connected by a network

Figure 1.48 Forty-nine computers connected by a network

Figure 1.49 Fifty computers connected by a network

Figure 1.50 Fifty-one computers connected by a network

Figure 1.51 Fifty-two computers connected by a network

Figure 1.52 Fifty-three computers connected by a network

Figure 1.53 Fifty-four computers connected by a network

Figure 1.54 Fifty-five computers connected by a network

Figure 1.55 Fifty-six computers connected by a network

Figure 1.56 Fifty-seven computers connected by a network

Figure 1.57 Fifty-eight computers connected by a network

Figure 1.58 Fifty-nine computers connected by a network

Figure 1.59 Sixty computers connected by a network

Figure 1.60 Sixty-one computers connected by a network

Figure 1.61 Sixty-two computers connected by a network

Figure 1.62 Sixty-three computers connected by a network

Figure 1.63 Sixty-four computers connected by a network

Figure 1.64 Sixty-five computers connected by a network

Figure 1.65 Sixty-six computers connected by a network

Figure 1.66 Sixty-seven computers connected by a network

Figure 1.67 Sixty-eight computers connected by a network

Figure 1.68 Sixty-nine computers connected by a network

Figure 1.69 Seventy computers connected by a network

Figure 1.70 Seventy-one computers connected by a network

Figure 1.71 Seventy-two computers connected by a network

Figure 1.72 Seventy-three computers connected by a network

Figure 1.73 Seventy-four computers connected by a network

Figure 1.74 Seventy-five computers connected by a network

Figure 1.75 Seventy-six computers connected by a network

Figure 1.76 Seventy-seven computers connected by a network

Figure 1.77 Seventy-eight computers connected by a network

Figure 1.78 Seventy-nine computers connected by a network

Figure 1.79 Eighty computers connected by a network

Figure 1.80 Eighty-one computers connected by a network

Figure 1.81 Eighty-two computers connected by a network

Figure 1.82 Eighty-three computers connected by a network

Figure 1.83 Eighty-four computers connected by a network

Figure 1.84 Eighty-five computers connected by a network

Figure 1.85 Eighty-six computers connected by a network

Figure 1.86 Eighty-seven computers connected by a network

Figure 1.87 Eighty-eight computers connected by a network

Figure 1.88 Eighty-nine computers connected by a network

Figure 1.89 Ninety computers connected by a network

Figure 1.90 Ninety-one computers connected by a network

Figure 1.91 Ninety-two computers connected by a network

Figure 1.92 Ninety-three computers connected by a network

Figure 1.93 Ninety-four computers connected by a network

Figure 1.94 Ninety-five computers connected by a network

Figure 1.95 Ninety-six computers connected by a network

Figure 1.96 Ninety-seven computers connected by a network

Figure 1.97 Ninety-eight computers connected by a network

Figure 1.98 Ninety-nine computers connected by a network

Figure 1.99 One hundred computers connected by a network

Figure 1.100 One hundred one computers connected by a network

Figure 1.101 One hundred two computers connected by a network

Figure 1.102 One hundred three computers connected by a network

Figure 1.103 One hundred four computers connected by a network

Figure 1.104 One hundred five computers connected by a network

Figure 1.105 One hundred six computers connected by a network

Figure 1.106 One hundred seven computers connected by a network

Figure 1.107 One hundred eight computers connected by a network

Figure 1.108 One hundred nine computers connected by a network

Figure 1.109 One hundred ten computers connected by a network

Figure 1.110 One hundred eleven computers connected by a network

Figure 1.111 One hundred twelve computers connected by a network

Figure 1.112 One hundred thirteen computers connected by a network

Figure 1.113 One hundred fourteen computers connected by a network

Figure 1.114 One hundred fifteen computers connected by a network

Figure 1.115 One hundred sixteen computers connected by a network

Figure 1.116 One hundred seventeen computers connected by a network

Figure 1.117 One hundred eighteen computers connected by a network

Figure 1.118 One hundred nineteen computers connected by a network

Figure 1.119 One hundred twenty computers connected by a network

Figure 1.120 One hundred twenty-one computers connected by a network

Figure 1.121 One hundred twenty-two computers connected by a network

Figure 1.122 One hundred twenty-three computers connected by a network

Figure 1.123 One hundred twenty-four computers connected by a network

Figure 1.124 One hundred twenty-five computers connected by a network

Figure 1.125 One hundred twenty-six computers connected by a network

Figure 1.126 One hundred twenty-seven computers connected by a network

Figure 1.127 One hundred twenty-eight computers connected by a network

Figure 1.128 One hundred twenty-nine computers connected by a network

Figure 1.129 One hundred thirty computers connected by a network

Figure 1.130 One hundred thirty-one computers connected by a network

Figure 1.131 One hundred thirty-two computers connected by a network

Figure 1.132 One hundred thirty-three computers connected by a network

Figure 1.133 One hundred thirty-four computers connected by a network

Figure 1.134 One hundred thirty-five computers connected by a network

Figure 1.135 One hundred thirty-six computers connected by a network

Figure 1.136 One hundred thirty-seven computers connected by a network

Figure 1.137 One hundred thirty-eight computers connected by a network

Figure 1.138 One hundred thirty-nine computers connected by a network

Figure 1.139 One hundred forty computers connected by a network

Figure 1.140 One hundred forty-one computers connected by a network

Figure 1.141 One hundred forty-two computers connected by a network

Figure 1.142 One hundred forty-three computers connected by a network

Figure 1.143 One hundred forty-four computers connected by a network

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(a) Any person who, without lawful authority, knowingly makes, publishes, or circulates any false statement or report concerning the financial condition of any bank, trust company, or savings association, with intent to injure the business of such institution, shall be guilty of a misdemeanor.

(b) Any person who, without lawful authority, knowingly makes, publishes, or circulates any false statement or report concerning the financial condition of any bank, trust company, or savings association, with intent to injure the business of such institution, shall be guilty of a misdemeanor.

(c) Any person who, without lawful authority, knowingly makes, publishes, or circulates any false statement or report concerning the financial condition of any bank, trust company, or savings association, with intent to injure the business of such institution, shall be guilty of a misdemeanor.

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(e) Any person who, without lawful authority, knowingly makes, publishes, or circulates any false statement or report concerning the financial condition of any bank, trust company, or savings association, with intent to injure the business of such institution, shall be guilty of a misdemeanor.

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(i) Any person who, without lawful authority, knowingly makes, publishes, or circulates any false statement or report concerning the financial condition of any bank, trust company, or savings association, with intent to injure the business of such institution, shall be guilty of a misdemeanor.

(j) Any person who, without lawful authority, knowingly makes, publishes, or circulates any false statement or report concerning the financial condition of any bank, trust company, or savings association, with intent to injure the business of such institution, shall be guilty of a misdemeanor.

Ans. Because the sum of the angles of a triangle is equal to two right angles.

Ques. Why is the sum of the angles of a triangle equal to two right angles?

Ans. Because the sum of the angles at the base of an isosceles triangle is equal to one right angle.

Ques. Why is the sum of the angles at the base of an isosceles triangle equal to one right angle?

Ans. Because the sum of the angles at the base of a right-angled isosceles triangle is equal to one right angle.

Ques. Why is the sum of the angles at the base of a right-angled

to the Lord Chief Justice of England, and to the Lord Chief Justice of Ireland, and to the Lord Chief Baron of the Exchequer, and to the Justices of the Queen's Bench, Common Pleas, and Exchequer, and to the Barons of the Exchequer in Ireland, and to the Master of the Rolls, and to the Vice-Chancellors, and to the Judges of the Court of Bankruptcy, and to the Judges of the County Courts, and to the Police Magistrates, and to the Clerks of the Peace, and to the Clerks of the Petty Sessions, and to the Sheriffs, and to the Coroners, and to the Justices of the Peace, and to the Commissioners of Bankrupts, and to the Commissioners of Insolvent Debtors, and to the Commissioners of Sewers, and to the Commissioners of Lieutenancy, and to the Commissioners of Customs, and to the Commissioners of Excise, and to the Commissioners of Stamps, and to the Commissioners of Taxes, and to the Commissioners of Woods, Forests, and Land Revenues, and to the Commissioners of Works, and to the Commissioners of Public Records, and to the Commissioners of Charitable Donations, and to the Commissioners of Inclosures, and to the Commissioners of Enrolments, and to the Commissioners of Audit, and to the Commissioners of Accounts, and to the Commissioners of Public Accounts, and to the Commissioners of Public Expenditure, and to the Commissioners of Public Revenue, and to the Commissioners of Public Debt, and to the Commissioners of Public Charities, and to the Commissioners of Public Education, and to the Commissioners of Public Health, and to the Commissioners of Public Works, and to the Commissioners of Public Buildings, and to the Commissioners of Public Parks, and to the Commissioners of Public Libraries, and to the Commissioners of Public Museums, and to the Commissioners of Public Galleries, and to the Commissioners of Public Monuments, and to the Commissioners of Public Statues, and to the Commissioners of Public Fountains, and to the Commissioners of Public Clocks, and to the Commissioners of Public Lamps, and to the Commissioners of Public Gas, and to the Commissioners of Public Water, and to the Commissioners of Public Sewers, and to the Commissioners of Public Drainage, and to the Commissioners of Public Roads, and to the Commissioners of Public Bridges, and to the Commissioners of Public Ferries, and to the Commissioners of Public Harbours, and to the Commissioners of Public Docks, and to the Commissioners of Public Piers, and to the Commissioners of Public Lighthouses, and to the Commissioners of Public Buoys, and to the Commissioners of Public Beacons, and to the Commissioners of Public Signals, and to the Commissioners of Public Telegraphs, and to the Commissioners of Public Railways, and to the Commissioners of Public Canals, and to the Commissioners of Public Navigation, and to the Commissioners of Public Shipping, and to the Commissioners of Public Seamen, and to the Commissioners of Public Fishing, and to the Commissioners of Public Oysters, and to the Commissioners of Public Whales, and to the Commissioners of Public Seals, and to the Commissioners of Public Walruses, and to the Commissioners of Public Polar Bears, and to the Commissioners of Public Penguins, and to the Commissioners of Public Albatrosses, and to the Commissioners of Public Gulls, and to the Commissioners of Public Terns, and to the Commissioners of Public Puffins, and to the Commissioners of Public Auks, and to the Commissioners of Public Guillemots, and to the Commissioners of Public Razorbills, and to the Commissioners of Public Kittiwakes, and to the Commissioners of Public Fulmars, and to the Commissioners of Public Petrels, and to the Commissioners of Public Shearwaters, and to the Commissioners of Public Gannets, and to the Commissioners of Public Cormorants, and to the Commissioners of Public Shags, and to the Commissioners of Public Pelicans, and to the Commissioners of Public Flamingos, and to the Commissioners of Public Herons, and to the Commissioners of Public Storks, and to the Commissioners of Public Ibises, and to the Commissioners of Public Spoonbills, and to the Commissioners of Public Curlews, and to the Commissioners of Public Godwits, and to the Commissioners of Public Snipes, and to the Commissioners of Public Woodcocks, and to the Commissioners of Public Sandpipers, and to the Commissioners of Public Plovers, and to the Commissioners of Public Lapwings, and to the Commissioners of Public Dotterels, and to the Commissioners of Public Turnstones, and to the Commissioners of Public Oystercatchers, and to the Commissioners of Public Avocets, and to the Commissioners of Public Stilts, and to the Commissioners of Public Phalaropes, and to the Commissioners of Public Rails, and to the Commissioners of Public Coots, and to the Commissioners of Public Moorhens, and to the Commissioners of Public Gallinules, and to the Commissioners of Public Crakes, and to the Commissioners of Public Corncrakes, and to the Commissioners of Public Bustards, and to the Commissioners of Public Cranes, and to the Commissioners of Public Herons, and to the Commissioners of Public Storks, and to the Commissioners of Public Ibises, and to the Commissioners of Public Spoonbills, and to the Commissioners of Public Curlews, and to the Commissioners of Public Godwits, and to the Commissioners of Public Snipes, and to the Commissioners of Public Woodcocks, and to the Commissioners of Public Sandpipers, and to the Commissioners of Public Plovers, and to the Commissioners of Public Lapwings, and to the Commissioners of Public Dotterels, and to the Commissioners of Public Turnstones, and to the Commissioners of Public Oystercatchers, and to the Commissioners of Public Avocets, and to the Commissioners of Public Stilts, and to the Commissioners of Public Phalaropes, and to the Commissioners of Public Rails, and to the Commissioners of Public Coots, and to the Commissioners of Public Moorhens, and to the Commissioners of Public Gallinules, and to the Commissioners of Public Crakes, and to the Commissioners of Public Corncrakes, and to the Commissioners of Public Bustards, and to the Commissioners of Public Cranes,

[illegible] [illegible] [illegible] [

the Lord Jesus Christ, and the love of God, and the communion of the Holy Ghost, be with you all. Amen.

2 Cor. xiii. 14.

Now unto Him that is able to keep you from falling, and to present you faultless before the presence of His glory with exceeding joy, to the only wise God our Saviour, be glory and majesty, dominion and power, both now and ever. Amen.

Jude 24, 25.

Now the God of peace that brought again from the dead our Lord Jesus, that great Shepherd of the sheep, through the blood of the everlasting covenant, make you perfect in every good work to do His will, working in you that which is well-pleasing in His sight, through Jesus Christ; to whom be glory for ever and ever. Amen.

Heb. xiii. 20, 21.

Now to Him that is able to do exceeding abundantly above all that we ask or think, according to the power that worketh in us, to Him be glory in the Church by Christ Jesus throughout all ages, world without end. Amen.

Eph. iii. 20, 21.

Now unto the King eternal, immortal, invisible, the only wise God, be honour and glory for ever and ever. Amen.

1 Tim. i. 17.

Now unto Him that is able to establish you according to my gospel, and the preaching of Jesus Christ, according to the revelation of the mystery, which was kept secret since the world began, but now is made manifest, and by the scriptures of the prophets, according to the commandment of the everlasting God, made known to all nations for the obedience of faith: to God only wise, be glory through Jesus Christ for ever. Amen.

Rom. xvi. 25-27.

Now unto God and our Father be glory for ever and ever. Amen.

Phil. iv. 20.

Now unto the King eternal, immortal, invisible, the only wise God, be honour and glory for ever and ever. Amen.

1 Tim. i. 17.

Now unto Him that is able to keep you from falling, and to present you faultless before the presence of His glory with exceeding joy, to the only wise God our Saviour, be glory and majesty, dominion and power, both now and ever. Amen.

Jude 24, 25

Mr. Lewis Cass, Jr., of Detroit, Michigan, was appointed Regent, in place of Mr. Ransom H. Gillet, deceased, and took his seat at the session of January, 1866.

Mr. Cass was born in Detroit, Michigan, in 1835, and graduated at the University of Michigan in 1856. He was appointed Secretary of Legation at Berlin, Prussia, in 1859, and remained there until 1861. He was appointed Secretary of Legation at Turin, Italy, in 1861, and remained there until 1865. He was appointed Secretary of Legation at Vienna, Austria, in 1865, and remained there until 1866. He was appointed Secretary of Legation at Paris, France, in 1866, and remained there until 1867. He was appointed Secretary of Legation at London, England, in 1867, and remained there until 1868. He was appointed Secretary of Legation at Berlin, Prussia, in 1868, and remained there until 1869. He was appointed Secretary of Legation at Turin, Italy, in 1869, and remained there until 1870. He was appointed Secretary of Legation at Vienna, Austria, in 1870, and remained there until 1871. He was appointed Secretary of Legation at Paris, France, in 1871, and remained there until 1872. 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He was appointed Secretary of Legation at London, England, in 2022, and remained there until 2023. He was appointed Secretary of Legation at Berlin, Prussia, in 2023, and remained there until 2024. He was appointed Secretary of Legation at Turin, Italy, in 2024, and remained there until 2025. He was appointed Secretary of Legation at Vienna, Austria, in 2025, and remained there until 2026. He was appointed Secretary of Legation at Paris, France, in 2026, and remained there until 2027. He was appointed Secretary of Legation at London, England, in 2027, and remained there until 2028. He was appointed Secretary of Legation at Berlin, Prussia, in 2028, and remained there until 2029. He was appointed Secretary of Legation at Turin, Italy, in 2029, and remained there until 2030. He was appointed Secretary of Legation at Vienna, Austria, in 2030, and remained there until 2031. He was appointed Secretary of Legation at Paris, France, in 2031, and remained there until 2032. He was appointed Secretary of Legation at London, England, in 2032, and remained there until 2033. He was appointed Secretary of Legation at Berlin, Prussia, in 2033, and remained there until 2034. He was appointed Secretary of Legation at Turin, Italy, in 2034, and remained there until 2035. He was appointed Secretary of Legation at Vienna, Austria, in 2035, and remained there until 2036. He was appointed Secretary of Legation at Paris, France, in 2036, and remained there until 2037. He was appointed Secretary of Legation at London, England, in 2037, and remained there until 2038. He was appointed Secretary of Legation at Berlin, Prussia, in 2038, and remained there until 2039. He was appointed Secretary of Legation at Turin, Italy, in 2039, and remained there until 2040. He was appointed Secretary of Legation at Vienna, Austria, in 2040, and remained there until 2041. He was appointed Secretary of Legation at Paris, France, in 2041, and remained there until 2042. 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He was appointed Secretary of Legation at London, England, in 2092, and remained there until 2093. He was appointed Secretary of Legation at Berlin, Prussia, in 2093, and remained there until 2094. He was appointed Secretary of Legation at Turin, Italy, in 2094, and remained there until 2095. He was appointed Secretary of Legation at Vienna, Austria, in 2095, and remained there until 2096. He was appointed Secretary of Legation at Paris, France, in 2096, and remained there until 2097. He was appointed Secretary of Legation at London, England, in 2097, and remained there until 2098. He was appointed Secretary of Legation at Berlin, Prussia, in 2098, and remained there until 2099. He was appointed Secretary of Legation at Turin, Italy, in 2099, and remained there until 2100. He was appointed Secretary of Legation at Vienna, Austria, in 2100, and remained there until 2101. He was appointed Secretary of Legation at Paris, France, in 2101, and remained there until 2102. He was appointed Secretary of Legation at London, England, in 2102, and remained there until 2103. He was appointed Secretary of Legation at Berlin, Prussia, in 2103, and remained there until 2104. He was appointed Secretary of Legation at Turin, Italy, in 2104, and remained there until 2105. He was appointed Secretary of Legation at Vienna, Austria, in 2105, and remained there until 2106. He was appointed Secretary of Legation at Paris, France, in 2106, and remained there until 2107. He was appointed Secretary of Legation at London, England, in 2107, and remained there until 2108. He was appointed Secretary of Legation at Berlin, Prussia, in 2108, and remained there until 2109. He was appointed Secretary of Legation at Turin, Italy, in 2109, and remained there until 2110. He was appointed Secretary of Legation at Vienna, Austria, in 2110, and remained there until 2111. He was appointed Secretary of Legation at Paris, France, in 2111, and remained there until 2112. He was appointed Secretary of Legation at London, England, in 2112, and remained there until 2113. He was appointed Secretary of Legation at Berlin, Prussia, in 2113, and remained there until 2114. He was appointed Secretary of Legation at Turin, Italy, in 2114, and remained there until 2115. He was appointed Secretary of Legation at Vienna, Austria, in 2115, and remained there until 2116. He was appointed Secretary of Legation at Paris, France, in 2116, and remained there until 2117. He was appointed Secretary of Legation at London, England, in 2117, and remained there until 2118. He was appointed Secretary of Legation at Berlin, Prussia, in 2118, and remained there until 2119. He was appointed Secretary of Legation at Turin, Italy, in 2119, and remained there until 2120. He was appointed Secretary of Legation at Vienna, Austria, in 2120, and remained there until 2121. He was appointed Secretary of Legation at Paris, France, in 2121, and remained there until 2122. He was appointed Secretary of Legation at London, England, in 2122, and remained there until 2123. He was appointed Secretary of Legation at Berlin, Prussia, in 2123, and remained there until 2124. He was appointed Secretary of Legation at Turin, Italy, in 2124, and remained there until 2125. He was appointed Secretary of Legation at Vienna, Austria, in 2125, and remained there until 2126. He was appointed Secretary of Legation at Paris, France, in 2126, and remained there until 2127. He was appointed Secretary of Legation at London, England, in 2127, and remained there until 2128. He was appointed Secretary of Legation at Berlin, Prussia, in 2128, and remained there until 2129. He was appointed Secretary of Legation at Turin, Italy, in 2129, and remained there until 2130. He was appointed Secretary of Legation at Vienna, Austria, in 2130, and remained there until 2131. He was appointed Secretary of Legation at Paris, France, in 2131, and remained there until 2132. He was appointed Secretary of Legation at London, England, in 2132, and remained there until 2133. He was appointed Secretary of Legation at Berlin, Prussia, in 2133, and remained there until 2134. He was appointed Secretary of Legation at Turin, Italy, in 2134, and remained there until 2135. He was appointed Secretary of Legation at Vienna, Austria, in 2135, and remained there until 2136. He was appointed Secretary of Legation at Paris, France, in 2136, and remained there until 2137. He was appointed Secretary of Legation at London, England, in 2137, and remained there until 2138. He was appointed Secretary of Legation at Berlin, Prussia, in 2138, and remained there until 2139. He was appointed Secretary of Legation at Turin, Italy, in 2139, and remained there until 2140. He was appointed Secretary of Legation at Vienna, Austria, in 2140, and remained there until 2141. He was appointed Secretary of Legation at Paris, France, in 2141, and remained there until 2142. He was appointed Secretary of Legation at London, England, in 2142, and remained there until 2143. He was appointed Secretary of Legation at Berlin, Prussia, in 2143, and remained there until 2144. He was appointed Secretary of Legation at Turin, Italy, in 2144, and remained there until 2145. He was appointed Secretary of Legation at Vienna, Austria, in 2145, and remained there until 2146. He was appointed Secretary of Legation at Paris, France, in 2146, and remained there until 2147. He was appointed Secretary of Legation at London, England, in 2147, and remained there until 2148. He was appointed Secretary of Legation at Berlin, Prussia, in 2148, and remained there until 2149. He was appointed Secretary of Legation at Turin, Italy, in 2149, and remained there until 2150. He was appointed Secretary of Legation at Vienna, Austria, in 2150, and remained there until 2151. He was appointed Secretary of Legation at Paris, France, in 2151, and remained there until 2152. He was appointed Secretary of Legation at London, England, in 2152, and remained there until 2153. He was appointed Secretary of Legation at Berlin, Prussia, in 2153, and remained there until 2154. He was appointed Secretary of Legation at Turin, Italy, in 2154, and remained there until 2155. He was appointed Secretary of Legation at Vienna, Austria, in 2155, and remained there until 2156. He was appointed Secretary of Legation at Paris, France, in 2156, and remained there until 2157. He was appointed Secretary of Legation at London, England, in 2157, and remained there until 2158. He was appointed Secretary of Legation at Berlin, Prussia, in 2158, and remained there until 2159. He was appointed Secretary of Legation at Turin, Italy, in 2159, and remained there until 2160. He was appointed Secretary of Legation at Vienna, Austria, in 2160, and remained there until 2161. He was appointed Secretary of Legation at Paris, France, in 2161, and remained there until 2162. He was appointed Secretary of Legation at London, England, in 2162, and remained there until 2163. He was appointed Secretary of Legation at Berlin, Prussia, in 2163, and remained there until 2164. He was appointed Secretary of Legation at Turin, Italy, in 2164, and remained there until 2165. He was appointed Secretary of Legation at Vienna, Austria, in 2165, and remained there until 2166. He was appointed Secretary of Legation at Paris, France, in 2166, and remained there until 2167. He was appointed Secretary of Legation at London, England, in 2167, and remained there until 2168. He was appointed Secretary of Legation at Berlin, Prussia, in 2168, and remained there until 2169. He was appointed Secretary of Legation at Turin, Italy, in 2169, and remained there until 2170. He was appointed Secretary of Legation at Vienna, Austria, in 2170, and remained there until 2171. He was appointed Secretary of Legation at Paris, France, in 2171, and remained there until 2172. He was appointed Secretary of Legation at London, England, in 2172, and remained there until 2173. He was appointed Secretary of Legation at Berlin, Prussia, in 2173, and remained there until 2174. He was appointed Secretary of Legation at Turin, Italy, in 2174, and remained there until 2175. He was appointed Secretary of Legation at Vienna, Austria, in 2175, and remained there until 2176. He was appointed Secretary of Legation at Paris, France, in 2176, and remained there until 2177. He was appointed Secretary of Legation at London, England, in 2177, and remained there until 2178. He was appointed Secretary of Legation at Berlin, Prussia, in 2178, and remained there until 2179. He was appointed Secretary of Legation at Turin, Italy, in 2179, and remained there until 2180. He was appointed Secretary of Legation at Vienna, Austria, in 2180, and remained there until 2181. He was appointed Secretary of Legation at Paris, France, in 2181, and remained there until 2182. He was appointed Secretary of Legation at London, England, in 2182, and remained there until 2183. He was appointed Secretary of Legation at Berlin, Prussia, in 2183, and remained there until 2184. He was appointed Secretary of Legation at Turin, Italy, in 2184, and remained there until 2185. He was appointed Secretary of Legation at Vienna, Austria, in 2185, and remained there until 2186. He was appointed Secretary of Legation at Paris, France, in 2186, and remained there until 2187. He was appointed Secretary of Legation at London, England, in 2187, and remained there until 2188. He was appointed Secretary of Legation at Berlin, Prussia, in 2188, and remained there until 2189. He was appointed Secretary of Legation at Turin, Italy, in 2189, and remained there until 2190. He was appointed Secretary of Legation at Vienna, Austria, in 2190, and remained there until 2191. He was appointed Secretary of Legation at Paris, France, in 2191, and remained there until 2192. He was appointed Secretary of Legation at London, England, in 2192, and remained there until 2193. He was appointed Secretary of Legation at Berlin, Prussia, in 2193, and remained there until 2194. He was appointed Secretary of Legation at Turin, Italy, in 2194, and remained there until 2195. He was appointed Secretary of Legation at Vienna, Austria, in 2195, and remained there until 2196. He was appointed Secretary of Legation at Paris, France, in 2196, and remained there until 2197. He was appointed Secretary of Legation at London, England, in 2197, and remained there until 2198. He was appointed Secretary of Legation at Berlin, Prussia, in 2198, and remained there until 2199. He was appointed Secretary of Legation at Turin, Italy, in 2199, and remained there until 2200. He was appointed Secretary of Legation at Vienna, Austria, in 2200, and remained there until 2201. He was appointed Secretary of Legation at Paris, France, in 2201, and remained there until 2202. He was appointed Secretary of Legation at London, England, in 2202, and remained there until 2203. He was appointed Secretary of Legation at Berlin, Prussia, in 2203, and remained there until 2204. He was appointed Secretary of Legation at Turin, Italy, in 2204, and remained there until 2205. He was appointed Secretary of Legation at Vienna, Austria, in 2205, and remained there until 2206. He was appointed Secretary of Legation at Paris, France, in 2206, and remained there until 2207. He was appointed Secretary of Legation at London, England, in 2207, and remained there until 2208. He was appointed Secretary of Legation at Berlin, Prussia, in 2208, and remained there until 2209. He was appointed Secretary of Legation at Turin, Italy, in 2209, and remained there until 2210. He was appointed Secretary of Legation at Vienna, Austria, in 2210, and remained there until 2211. He was appointed Secretary of Legation at Paris, France, in 2211, and remained there until 2212. He was appointed Secretary of Legation at London, England, in 2212, and remained there until 2213. He was appointed Secretary of Legation at Berlin, Prussia, in 2213, and remained there until 2214. He was appointed Secretary of Legation at Turin, Italy, in 2214, and remained there until 2215. He was appointed Secretary of Legation at Vienna, Austria, in 2215, and remained there until 2216. He was appointed Secretary of Legation at Paris, France, in 2216, and remained there until 2217. He was appointed Secretary of Legation at London, England, in 2217, and remained there until 2218. He was appointed Secretary of Legation at Berlin, Prussia, in 2218, and remained there until 2219. He was appointed Secretary of Legation at Turin, Italy, in 2219, and remained there until 2220. He was appointed Secretary of Legation at Vienna, Austria, in 2220, and remained there until 2221. He was appointed Secretary of Legation at Paris, France, in 2221, and remained there until 2222. He was appointed Secretary of Legation at London, England, in 2222, and remained there until 2223. He was appointed Secretary of Legation at Berlin, Prussia, in 2223, and remained there until 2224. He was appointed Secretary of Legation at Turin, Italy, in 2224, and remained there until 2225. He was appointed Secretary of Legation at Vienna, Austria, in 2225, and remained there until 2226. He was appointed Secretary of Legation at Paris, France, in 2226, and remained there until 2227. He was appointed Secretary of Legation at London, England, in 2227, and remained there until 2228. He was appointed Secretary of Legation at Berlin, Prussia, in 2228, and remained there until 2229. He was appointed Secretary of Legation at Turin, Italy, in 2229, and remained there until 2230. He was appointed Secretary of Legation at Vienna, Austria, in 2230, and remained there until 2231. He was appointed Secretary of Legation at Paris, France, in 2231, and remained there until 2232. He was appointed Secretary of Legation at London, England, in 2232, and remained there until 2233. He was appointed Secretary of Legation at Berlin, Prussia, in 2233, and remained there until 2234. He was appointed Secretary of Legation at Turin, Italy, in 2234, and remained there until 2235. He was appointed Secretary of Legation at Vienna, Austria, in 2235, and remained there until 2236. He was appointed Secretary of Legation at Paris, France, in 2236, and remained there until 2237. He was appointed Secretary of Legation at London, England, in 2237, and remained there until 2238. He was appointed Secretary of Legation at Berlin, Prussia, in 2238, and remained there until 2239. He was appointed Secretary of Legation at Turin, Italy, in 2239, and remained there until 2240. He was appointed Secretary of Legation at Vienna, Austria, in 2240, and remained there until 2241. He was appointed Secretary of Legation at Paris, France, in 2241, and remained there until 2242. He was appointed Secretary of Legation at London, England, in 2242, and remained there until 2243. He was appointed Secretary of Legation at Berlin, Prussia, in 2243, and remained there until 2244. He was appointed Secretary of Legation at Turin, Italy, in 2244, and remained there until 2245. He was appointed Secretary of Legation at Vienna, Austria, in 2245, and remained there until 2246. He was appointed Secretary of Legation at Paris, France, in 2246, and remained there until 2247. He was appointed Secretary of Legation at London, England, in 2247, and remained there until 2248. He was appointed Secretary of Legation at Berlin, Prussia, in 2248, and remained there until 2249. He was appointed Secretary of Legation at Turin, Italy, in 2249, and remained there until 2250. He was appointed Secretary of Legation at Vienna, Austria, in 2250, and remained there until 2251. He was appointed Secretary of Legation at Paris, France, in 2251, and remained there until 2252. He was appointed Secretary of Legation at London, England, in 2252, and remained there until 2253. He was appointed Secretary of Legation at Berlin, Prussia, in 2253, and remained there until 2254. He was appointed Secretary of Legation at Turin, Italy, in 2254, and remained there until 2255. He was appointed Secretary of Legation at Vienna, Austria, in 2255, and remained there until 2256. He was appointed Secretary of Legation at Paris, France, in 2256, and remained there until 2257. He was appointed Secretary of Legation at London, England, in 2257, and remained there until 2258. He was appointed Secretary of Legation at Berlin, Prussia, in 2258, and remained there until 2259. He was appointed Secretary of Legation at Turin, Italy, in 2259, and remained there until 2260. He was appointed Secretary of Legation at Vienna, Austria, in 2260, and remained there until 2261. He was appointed Secretary of Legation at Paris, France, in 2261, and remained there until 2262. He was appointed Secretary of Legation at London, England, in 2262, and remained there until 2263. He was appointed Secretary of Legation at Berlin, Prussia, in 2263, and remained there until 2264. He was appointed Secretary of Legation at Turin, Italy, in 2264, and remained there until 2265. He was appointed Secretary of Legation at Vienna, Austria, in 2265, and remained there until 2266. He was appointed Secretary of Legation at Paris, France, in 2266, and remained there until 2267. He was appointed Secretary of Legation at London, England, in 2267, and remained there until 2268. He was appointed Secretary of Legation at Berlin, Prussia, in 2268, and remained there until 2269. He was appointed Secretary of Legation at Turin, Italy, in 2269, and remained there until 2270. He was appointed Secretary of Legation at Vienna, Austria, in 2270, and remained there until 2271. He was appointed Secretary of Legation at Paris, France, in 2271, and remained there until 2272. He was appointed Secretary of Legation at London, England, in 2272, and remained there until 2273. He was appointed Secretary of Legation at Berlin, Prussia, in 2273, and remained there until 2274. He was appointed Secretary of Legation at Turin, Italy, in 2274, and remained there until 2275. He was appointed Secretary of Legation at Vienna, Austria, in 2275, and remained there until 2276. He was appointed Secretary of Legation at Paris, France, in 2276, and remained there until 2277. He was appointed Secretary of Legation at London, England, in 2277, and remained there until 2278. He was appointed Secretary of Legation at Berlin, Prussia, in 2278, and remained there until 2279. He was appointed Secretary of Legation at Turin, Italy, in 2279, and remained there until 2280. He was appointed Secretary of Legation at Vienna, Austria, in 2280, and remained there until 2281. He was appointed Secretary of Legation at Paris, France, in 2281, and remained there until 2282. He was appointed Secretary of Legation at London, England, in 2282, and remained there until 2283. He was appointed Secretary of Legation at Berlin, Prussia, in 2283, and remained there until 2284. He was appointed Secretary of Legation at Turin, Italy, in 2284, and remained there until 2285. He was appointed Secretary of Legation at Vienna, Austria, in 2285, and remained there until 2286. He was appointed Secretary of Legation at Paris, France, in 2286, and remained there until 2287. He was appointed Secretary of Legation at London, England, in 2287, and remained there until 2288. He was appointed Secretary of Legation at Berlin, Prussia, in 2288, and remained there until 2289. He was appointed Secretary of Legation at Turin, Italy, in 2289, and remained there until 2290. He was appointed Secretary of Legation at Vienna, Austria, in 2290, and remained there until 2291. He was appointed Secretary of Legation at Paris, France, in 2291, and remained there until 2292. He was appointed Secretary of Legation at London, England, in 2292, and remained there until 2293. He was appointed Secretary of Legation at Berlin, Prussia, in 2293, and remained there until 2294. He was appointed Secretary of Legation at Turin, Italy, in 2294, and remained there until 2295. He was appointed Secretary of Legation at Vienna, Austria, in 2295, and remained there until 2296. He was appointed Secretary of Legation at Paris, France, in 2296, and remained there until 2297. He was appointed Secretary of Legation at London, England, in 2297, and remained there until 2298. He was appointed Secretary of Legation at Berlin, Prussia, in 2298, and remained there until 2299. He was appointed Secretary of Legation at Turin, Italy, in 2299, and remained there until 2300. He was appointed Secretary of Legation at Vienna, Austria, in 2300, and remained there until 2301. He was appointed Secretary of Legation at Paris, France, in 2301, and remained there until 2302. He was appointed Secretary of Legation at London, England, in 2302, and remained there until 2303. He was appointed Secretary of Legation at Berlin, Prussia, in 2303, and remained there until 2304. He was appointed Secretary of Legation at Turin, Italy, in 2304, and remained there until 2305. He was appointed Secretary of Legation at Vienna, Austria, in 2305, and remained there until 2306. He was appointed Secretary of Legation at Paris, France, in 2306, and remained there until 2307. He was appointed Secretary of Legation at London, England, in 2307, and remained there until 2308. He was appointed Secretary of Legation at Berlin, Prussia, in 2308, and remained there until 2309. He was appointed Secretary of Legation at Turin, Italy, in 2309, and remained there until